

Golf Ball Flight Dynamics

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Abstract

Projectile motion is a field of study that has many practical applications. In this case the projection of a golf ball will be analyzed. The golf balls projection is effected by many different forces, here the spin and drag will be the main areas of concern. A system of differential equations will be interpreted from aerodynamics and drag to aid in the discussion. Specifically, the magnus effect and resistance due to aerodynamic drag are considered.

1 Introduction

Golf balls didn't all ways have dimples on them. The dimpled golf ball was first introduced in 1905 by William Taylor, an engineer. He found that dimpled balls were more aerodynamic, allowing golfers to have better control and further drives. The number of dimples on a typical golf ball ranges from 250-450. Each manufacturer has a different theory on the number of dimples, depth of dimple and in the pattern in which the dimples are placed. All these attributes contribute to the balls overall aerodynamic qualities.

Today, golf ball manufacturers are trying to engineer the perfect golf ball. The United States Golf Association (USGA) strictly regulates the design and material used in golf ball manufacturing. With the use of modern material to create different cores a designer has the ability to create a ball that has different properties such as greater spin, soft feel at impact and lower trajectory. For these reasons the manufacturing of golf balls is highly regulated.

2 Our Model

A golf ball's flight pattern is affected directly by the presence of air resistance and the rotation of the ball. Looking at both air resistance and rotation flight paths can be predicted. In this model both air resistance and rotation are used to set up a system of differential equations that can be used to predict several different flight paths. Matlab was used to solve the system of equations using a numerical solver. The calculated trajectories were then plotted to show the effects of angular acceleration and aerodynamic drag. One system is used to model a smooth ball while another is used to model a dimpled ball. Throughout the models the initial velocity and angle of attack affect the trajectories. Initial velocity is the velocity the ball leaves the $x - y$ plane. Angle of attack is the angle between the $x - y$ plane and initial linear velocity.

3 Classic Parabolic Motion

First, a discussion in classic parabolic motion is in order to acquire the knowledge needed to understand the golf balls flight. classic parabolic motion can be derived from a system of two differential equations, in the x and y directions. In the classical model, every force, excluding gravity, is neglected. This assumes no wind, drag, magnus, change in altitude, or changes in humidity. From Newton's second law of motion, and the fact that gravity is the only force acting on the ball,

$$\begin{aligned}\sum F_y &= my'' \\ my'' &= -mg \\ y'' &= -g \\ y' &= -gt + C_1 \\ y &= -\frac{1}{2}gt^2 + C_1t + C_2\end{aligned}$$

Solving for C_1 and C_2 yields

$$\begin{aligned}C_1 &= V_{0y} \\ C_2 &= y_0\end{aligned}$$

and

$$y = -\frac{1}{2}gt^2 + V_{0y}t + y_0$$

In the x direction, no forces act on the ball.

$$\begin{aligned}\sum F_x &= mx'' \\ mx'' &= 0 \\ x'' &= 0 \\ x' &= C_3 \\ x &= C_3t + C_4\end{aligned}$$

solving for C_3 and C_4 yields

$$\begin{aligned}C_3 &= V_{0x} \\ C_4 &= X_0\end{aligned}$$

and

$$x = V_{0x}t + X_0$$

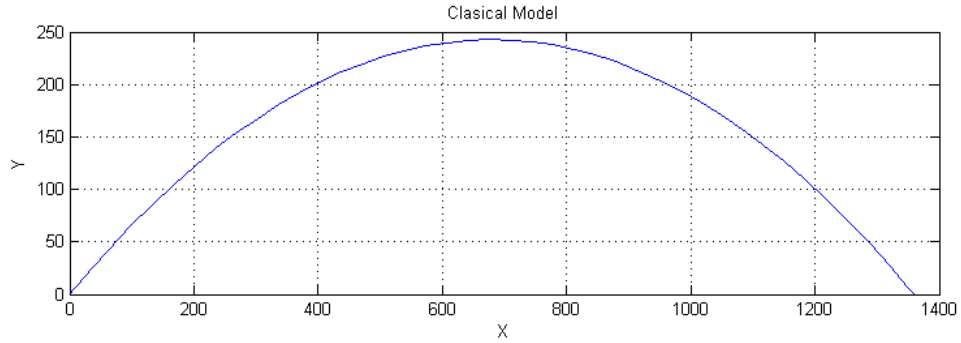


Figure 1: Classic Parabolic Motion

From this equation it is easy to see that the velocity of the ball in the x direction stays constant throughout the duration of the flight. Note that throughout these systems, the initial coordinates of the ball will be $(0, 0)$

4 General Equations

From Newtons second law of motion the flight equations were derived for the trajectories of the ball. The equations were derived using air resistance and the Magnus Force. the flight equations were in three directions.

$$\sum F_y = my'' = -Weight - AirResistance + MagnusForce$$

$$\sum F_x = mx'' = -AirResistance + MagnusForce$$

$$\sum F_z = mz'' = -AirResistance + MagnusForce$$

5 Air Resistance

The force put on the golf ball due to its interaction with air molecules is called aerodynamic drag or air resistance. The air resistance can be calculated using the equation shown below,

$$R = \frac{1}{2}C_D\rho AV^2$$

where,

C_D = Drag Coefficient

ρ = Air density

A = Surface Area

V = Velocity

The air resistance must be considered in all three directions:

$$R_x = \frac{1}{2}C_d\rho AV_x^2$$

$$R_y = \frac{1}{2}C_d\rho AV_y^2$$

$$R_z = \frac{1}{2}C_d\rho AV_z^2$$

6 Smooth Ball

Lets think of a smooth ball such as a ping pong ball for this model. When a ball travels though the air there is resistance due to the contact between the surface of the ball and the air molecules. The air surrounding the ball as it flies is called the boundary layer. In this case, the air molecules tend to pass by the ball and around the ball in a parallel fashion this is laminar flow. The laminar flow creates separation around the ball creating a large wake of low pressure behind the ball, decelerating it. Only air resistance is considered in this case.

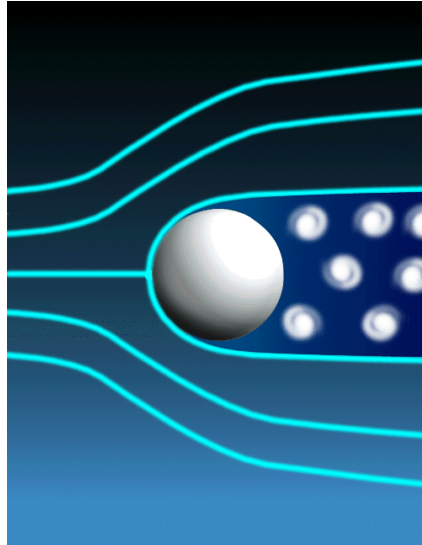


Figure 2: Laminar Flow

6.1 Assumptions

The following assumptions were made to simplify the smooth ball model.

1. No Spin
2. No wind
3. No changes in air conditions due to altitude or humidity
4. No change in Barometric pressure

R = Air Resistance

m = Mass

V_y = Velocity in Y

V_x = Velocity in X

V_z = Velocity in Z

6.2 Flight Equations

$$\sum F_y = my'' = -mg - R(V_y)$$

$$\sum F_x = mx'' = -R(V_x)$$

$$\sum F_z = mz'' = -R(V_z)$$

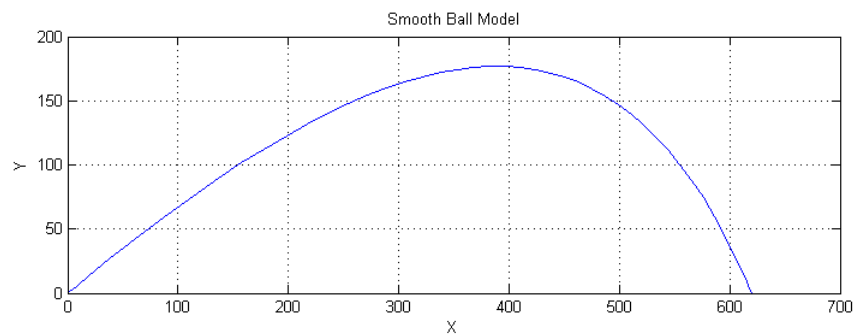


Figure 3: Smooth Ball Projection

7 Magnus Force

Have you ever notice that then you throw a baseball or hit a golf ball the flight pattern is much different when you place spin on the ball. When spin is placed on a ball the pressure around the ball is altered, this is called the magnus effect. Lets look at a backspin and how it effects the flight of a golf ball. When backspin is placed on a golf ball it will allow the ball to gain loft and have a longer flight. When the ball spins it directs air upwards in front and downward behind it, creating a pressure difference. The pressure difference pushes the ball upward allowing the ball to stay in flight for greater time, this is called the Magnus Force.

7.1 Magnus Formula

The Magnus force lifting the ball is caused by the rotation and the velocity of the ball. The rotation causes air disturbances perpendicular to the direction of the angular velocity causing the direction of flight is altered perpendicular to the linear velocity. This relationship shows that the Magnus force is calculated using a cross product.

$$\mathbf{F}_M = S(\omega \times V)$$

V =Velocity

ω =Angular Velocity

S =Magnus Coefficient

The cross-product provides *Magnus Force* in the three dimensions

$$S \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_i & \omega_j & \omega_k \\ V_i & V_j & V_k \end{vmatrix} = S[(\omega_j V_k - \omega_i V_j)\mathbf{i} - (\omega_i V_k - \omega_k V_i)\mathbf{j} + (\omega_i V_j - \omega_j V_i)\mathbf{k}]$$

$$\mathbf{F}_{Mx} = S(\omega_j V_k - \omega_k V_j)$$

$$\mathbf{F}_{My} = S(\omega_k V_i - \omega_i V_k)$$

$$\mathbf{F}_{Mz} = S(\omega_i V_j - \omega_j V_i)$$

8 Dimpled Ball

As discussed in the smooth ball model, when a ball travels through the air a disturbance is produced. The difference between the smooth ball and the dimpled ball is this disturbance. The boundary layer surrounding a dimpled ball is turbulent. This turbulent flow is able to stay very close to the surface of the ball for a longer period of time than a laminar flow. The turbulent flow allows for a smaller separation of air flow around the ball creating a smaller area of low pressure behind the ball, causing deceleration at a slower rate than the smooth ball. The magnitude of the Magnus force is drastically increased with the dimples, therefore both the Magnus Force and air resistance are considered in this case.

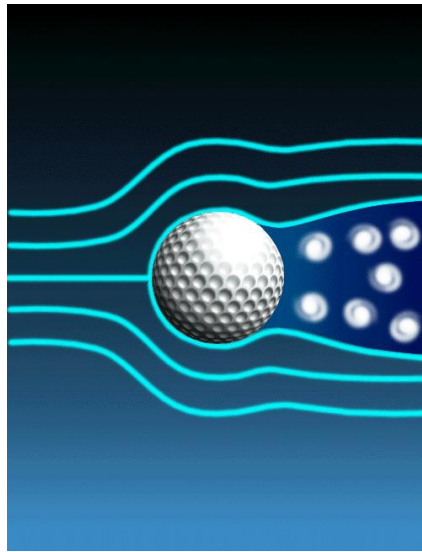


Figure 4: Turbulent Flow

8.1 Assumptions

The following assumptions were made to simplify the dimpled ball model.

1. No wind
2. No change in air conditions due to altitude or humidity
3. No changes in Barometric pressure

R =Air Resistance

F_M =Magus Force

ω =Omega

m =Mass

V_y =Velocity in Y

V_x =Velocity in X

V_z =Velocity in Z

8.2 Flight Equations

The flight equations make up the system of equations that govern the trajectory of the golf ball. The equations account for air resistance and magnus force.

$$\sum F_y = my'' = -mg - R(V_y) + F_M(\omega, v_x, v_z,)$$

$$y'' = -g - \frac{R}{m}(V_y) + \frac{F_M}{m}(\omega, v_x, v_z)$$

$$\sum F_x = mx'' = -R(V_x) + F_M(\omega, v_y, v_z,)$$

$$x'' = -\frac{R}{m}(V_x) + \frac{F_M}{m}(\omega, v_y, v_z)$$

$$\sum F_z = mz'' = -R(V_z) + F_M(\omega, v_x, v_y,)$$

$$z'' = -\frac{R}{m}(V_z) + \frac{F_M}{m}(\omega, v_x, v_y)$$

8.3 Backspin

These figures use the same initial velocity and angle of attack.

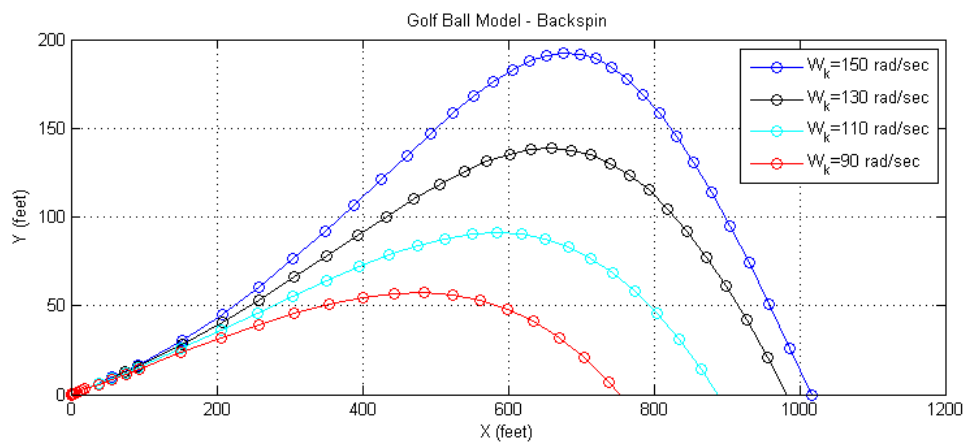


Figure 5: Backspin Models

8.4 Hook and Slice

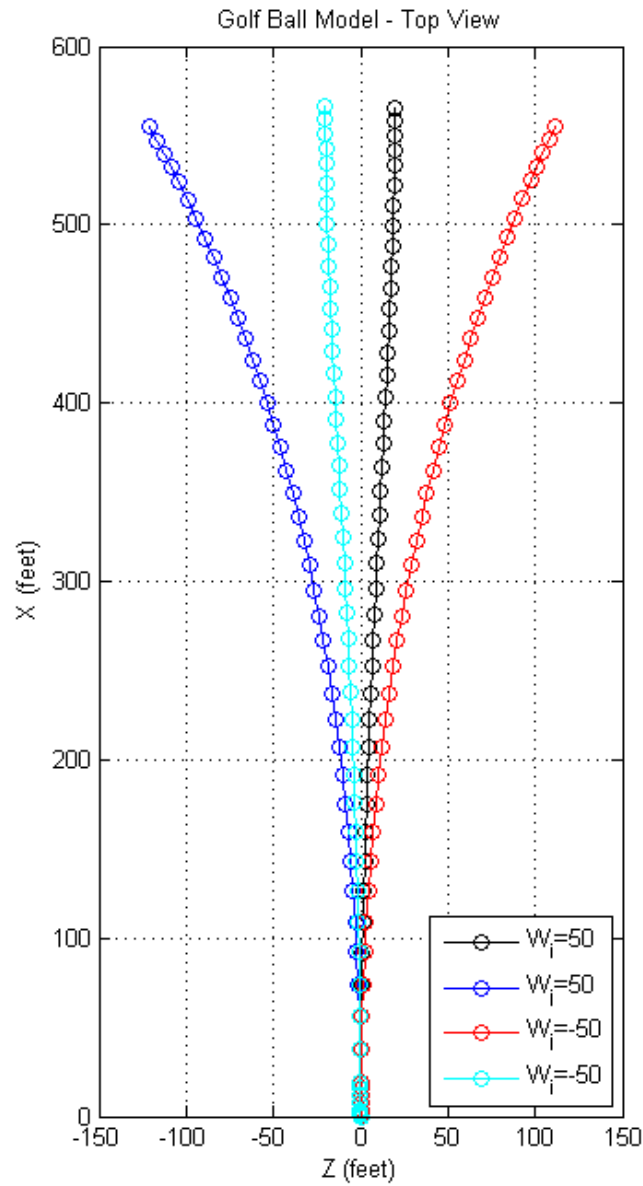


Figure 6: Hook, Pull, Push, and Slice, from left to right

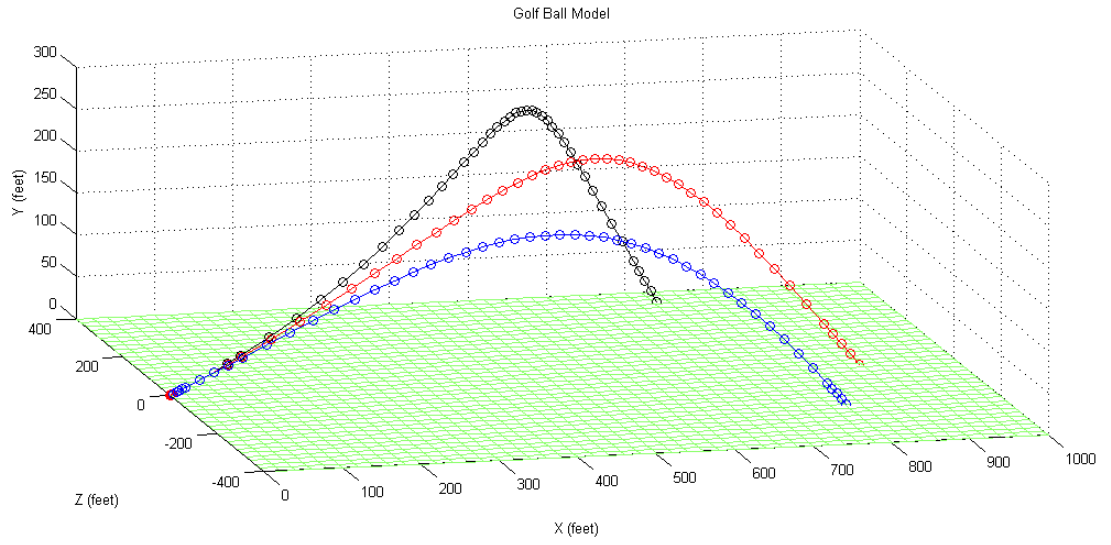


Figure 7: 3-dimensional view. These figures use the same initial velocity and angle of attack.

8.5 Comparison

These figures use the same initial velocity and angle of attack. There is backspin (ω_k) of 130 radians per second on the dimpled ball trajectory. Notice how the dimpled golf ball flies more than twice the distance of the smooth ball. This is caused by the reduced coefficient of drag due to the turbulent flow behind the dimpled ball.

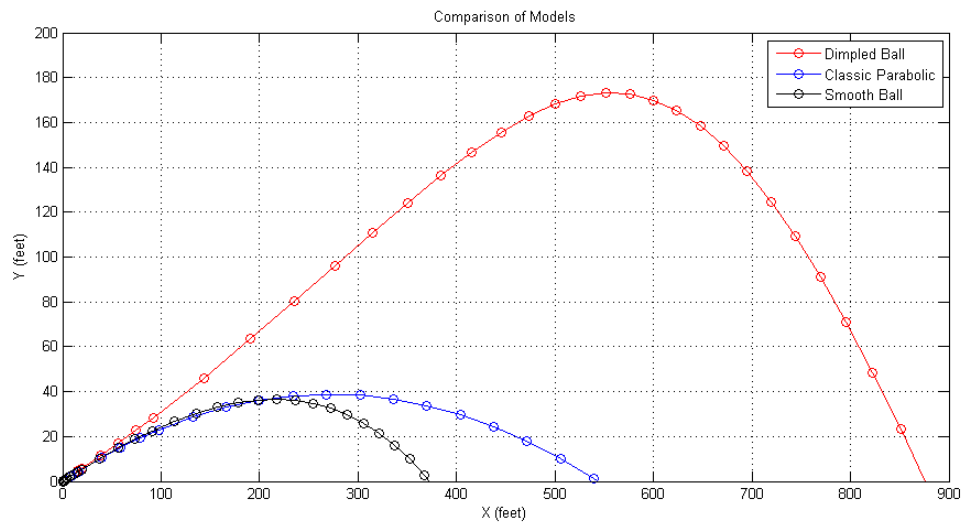


Figure 8: Comparing the Models

9 Conclusion

It is quit clear to see the effect dimples have on a golf ball trajectory and control. The increase in control and trajectory is directly related to the turbulent boundary layer that is created by the dimples. The smooth balls trajectory is about half a far as the trajectory of a golf ball. The systems of equations that were used are very simple, only taking into account ball rotation and drag. The dimpled ball system accounted for the effects of dimples but for further analysis a greater knowledge in aerodynamic theory is required.

10 References

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11 Appendix

11.1 Classic Parabolic Motion m-file

```
function Classicball
%initial velocity = 215.06 feet/sec = 146.63 m/h @ 35.54 degrees
[t,x]=ode45(@flights,[0,7.763],[0,175,0,125,0,0]);
plot(x(:,1),x(:,3));
title('Clasical Model')
xlabel('X')
ylabel('Y')
grid on
end
function xprime=flights(t,x)
xprime=zeros(2,1);
%X
xprime(1)=x(2);
xprime(2)=0;
%Y
xprime(3)=x(4);
xprime(4)=-32.2;
%Z
xprime(5)=x(6);
xprime(6)=0;
end$
```

11.2 Smoothball

```
$function Smoothball
[t,x]=ode45(@flights,[0,6.299],[0,175,0,125,0,0]);
%initial velocity = 215.06 feet/sec = 146.63 m/h @ 35.54 degrees
plot(x(:,1),x(:,3));
title('Smooth Ball Model')
xlabel('X')
ylabel('Y')
grid on
end
function xprime=flights(t,x)
```

```

%Parameters
C_d=.25;
r=.002378;
A=.25*pi*(1.75/12)^2;
m=(1.5/(16*32.2));
%D =((1/2)*C_d*r*A*V^2
%Square of the velocity is in the equation, therefor
D =((1/2)*C_d*r*A);
xprime=zeros(2,1);
%X
xprime(1)=x(2);
xprime(2)=- (D/m)*x(2)^2;
%Y
xprime(3)=x(4);
xprime(4)=-32.2-(D/m)*x(4)^2;
%Z
xprime(5)=x(6);
xprime(6)=- (D/m)*x(6)^2;
end$

```

11.3 3-dimensional Dimpled Ball Models

```

$function Golfball
[t,x]=ode45(@flightg,[0,8.3],[0,175,0,75,0,0]);
plot3(x(:,1),x(:,5),x(:,3),'-ob');
title('Golf Ball Model')
xlabel('X (feet)')
ylabel('Z (feet)')
zlabel('Y (feet)')
grid on
hold on
end
function xprime=flightg(t,x)
%Parameters
%Drag
C_d=.15;
r=.002378;
A=.25*pi*(1.75/12)^2;

```

```

m=(1.5/(16*32.2));
%D =((1/2)*C_d*r*A*V^2
%Square of the velocity is in the equation, therefor
D =((1/2)*C_d*r*A);
%Magnus
s=.000005;
m=(1.5/(16*32.2));
M=(s/m);
W_I=-100;
W_J=0;
W_K=110;
xprime=zeros(2,1);
%X
xprime(1)=x(2);
xprime(2)=- (D/m)*x(2)^2+M*(W_J*x(6)-W_K*x(4));
%Y
xprime(3)=x(4);
xprime(4)=-32.2-(D/m)*x(4)^2+M*(W_K*x(2)-W_I*x(6));
%Z
xprime(5)=x(6);
xprime(6)=- (D/m)*x(6)^2+M*(W_I*x(4)-W_J*x(2));
end$

```