Union College Math Conference: Stochastic Analysis and Applications

June 3–5, 2022

Friday program

3:15–4:30pm: Session I (ISEC 187)

- 3:15–3:30 Welcome and opening remarks
- 3:30–3:55 Fabrice Baudoin: *Asymptotic windings of Brownian motions on unitary groups*
- 4:00–4:25 Li Chen: *Dirichlet fractional Gaussian fields on the Sierpinski gasket*

4:30–5:30pm: Reception (Olin Rotunda)

Saturday program

7:30–8:00am: Coffee & pastries, registration (Olin Rotunda)

8:00–9:00am: Plenary talk (Olin 115)
Tai Melcher: *Large deviations for sub-Riemannian random walks*

9:00–10:30am: Session II (ISEC 222)

- 9:00–9:25 Raluca Balan: *Spatial integral of the solution to SPDEs with time-independent noise*
- 9:30–9:55 Robert Neel: *Localization and asymptotics of logarithmic derivatives of the heat kernel in small time*
- 10:00–10:25 Vincent R Martinez: *On ergodicity of the damped-driven stochastically forced KdV equation*

10:30–11:00am: Coffee break (Olin Rotunda)

11:00am–12:00pm: Plenary talk (Olin 115)

12:00–2:00pm: Lunch break

2:00–4:00pm: Session III (ISEC 222)

- 2:00–2:25 Pratima Hebbar: *Limit theorems for branching diffusion processes*
- 2:30–2:55 Jina Hyoungji Kim: *From finite to infinite dimensions: functional inequalities for linear diffusions with degenerate noise*
- 3:00–3:25 Jinwoo Sung: *Minkowski content of Liouville quantum gravity metric spaces*
- 3:30–3:55 Jing Wang: *Spectral bounds for exit times of diffusions on metric measure spaces*

4:00–4:30pm: Coffee break (Olin Rotunda)

4:30–5:30pm: Plenary talk (Olin 115)

Sunday program

8:00–8:30am: Coffee & pastries (Olin Rotunda)

8:30–11:00am: Session IV (ISEC 222)

- 8:30–8:55 Evan Camrud: *Exponential convergence of the degenerate stochastic Lorenz 96 model*
- 9:00–9:25 Liangbing Luo: *Logarithmic Sobolev inequalities on non-isotropic Heisenberg groups*
- 9:30–9:55 Nathaniel Eldredge: *Dual functional inequalities and optimal transport*
- 10:00–10:25 Dalton A R Sakthivadivel: *Statistical Inference and the Parallel Transport of Probability*
- 10:30–11:00 Wenjian Liu: *Split canonical relations and geometric quantization*

11:00–11:30am: Coffee break (Olin Rotunda)

11:30am–12:30pm: Plenary talk (Olin 115)

12:30–2:00pm: Lunch break

2:00–3:00pm: Session V (ISEC 222)

- 2:00–2:25 Xiaoming Song: *Spatial averages for the Parabolic Anderson model driven by rough noise*
- 2:30–3:00 Ioannis Gasteratos: *Importance sampling for stochastic reaction-diffusion equations in the moderate deviation regime*

Abstracts

Tai Melcher (University of Virginia)

Plenary talk: *Large deviations for sub-Riemannian random walks*

One of the first important theorems one learns in probability is the law of large numbers, a primitive version of which says that, for S_n the number of heads in n tosses of a fair coin, the relative frequency of heads S_n/n converges to 1/2. The process S_n is an example of a random walk on R. A large deviations principle quantifies this convergence by finding the asymptotic rate of decay of the probability of events like S_n/n being larger than 1/2 for large values of n. More generally, large deviations techniques are used to study the exponential rate of decay of probabilities of increasingly unlikely events. These results have applications, for example, in statistical mechanics, information theory, and insurance.

In this talk we will discuss large deviations results for random walks on stratified nilpotent Lie groups. For such groups, there is a collection of vectors generating the Lie algebra, which equips the group with a natural but degenerate geometry. We consider random walks with increments in only these directions and show that, under certain constraints on the distribution of the increments, a large deviation principle holds with a natural rate function adapted to the subRiemannian geometry of these spaces.

This is joint work with Jing Wang and Masha Gordina.

Raluca Balan (University of Ottawa)

Spatial integral of the solution to SPDEs with time-independent noise

In this talk, we review some recent results regarding the asymptotic behavior of the spatial integral of the solution to the hyperbolic/parabolic Anderson model, as the domain of the integral gets large (for fixed time t). This equation is driven by a spatially homogeneous Gaussian noise. The noise does not depend on time, which means that Itô's martingale theory for stochastic integration cannot be used. Using the methodology initiated in Huang, Nualart and Viitasaari (2020), which consists of a combination of Malliavin calculus techniques with Stein's method for normal approximations, we show that with proper normalization and centering, the spatial integral of the solution converges to a standard normal distribution, by estimating the speed of this convergence in the total variation distance. This talk is based on joint work with Wangjun Yuan.

Fabrice Baudoin (University of Connecticut)

Asymptotic windings of Brownian motions on unitary groups

We study the Brownian motion on the non-compact Grassmann manifold and some of its functionals. The key point is to realize this Brownian motion as a matrix diffusion process, use matrix stochastic calculus and take advantage of the hyperbolic Stiefel fibration to study a functional that can be understood in that setting as a generalized stochastic area process. In particular, an application to the study of Brownian windings in the Lie group $U(n-k,k)$ are then given. This is joint work with Nizar Demni (University of Marseille) and Jing Wang (Purdue University).

Li Chen (Louisiana State University)

Dirichlet fractional Gaussian fields on the Sierpinski gasket

In this talk, we discuss the Dirichlet fractional Gaussian fields on the Sierpinski gasket. We show that they are limits of fractional discrete Gaussian fields defined on the sequence of canonical approximating graphs. This is a joint work with Fabrice Baudoin (UConn).

Evan Camrud (Iowa State University/Colorado State University)
Exponential convergence of the degenerate stochastic Lorenz 96 model
It is a quick exercise to prove that a stochastic Lorenz 96 model with nondegenerate dissipation converges to its unique invariant probability measure exponentially fast. When the dissipation is degenerate, however, the standard Lyapunov approach (with the infinitesimal generator of the process) is no longer valid. We establish exponential convergence to the unique invariant measure in the degenerate setting by computing pathwise estimates of the system and showing that, in expectation and over positive time scales, the dissipation exceeds a fraction of the initial energy.

Nathaniel Eldredge (University of Northern Colorado)
Dual functional inequalities and optimal transport
Many important results in stochastic analysis take the form of functional inequalities for the transition semigroup P_t of a continuous-time Markov process, such as Brownian motion. They are usually stated in terms of the action $P_t f$ of the semigroup upon functions on the state space X. However, the semigroup P_t has a dual action μP_t on the space $\mathcal{P}(X)$ of probability measures, and so one may look for corresponding dual formulations of useful functional inequalities. Often they take the form of a Lipschitz condition for P_t with respect to appropriate optimal transport distances on $\mathcal{P}(X)$. Building on previous work of Kuwada and Luise–Savaré, we obtain equivalent dual formulations for the reverse Poincaré and reverse logarithmic Sobolev inequalities, in a very general setting that is free of geometric assumptions.

This is joint work with Fabrice Baudoin.

Ioannis Gasteratos (Imperial College)
Importance sampling for stochastic reaction-diffusion equations in the moderate deviation regime Rare events of significant importance arise in several scientific fields including climate modeling, chemistry, material science and quantum mechanics. In this talk, we are concerned with the development of numerical methods for the simulation of rare events for Stochastic Reaction-Diffusion Equations (SRDEs) in the range of moderate deviations. The latter describes an asymptotic regime that interpolates between the Central Limit Theorem and Large Deviation Principle. We develop a provably efficient importance sampling scheme that estimates exit probabilities of solutions to small-noise SRDEs from scaled neighborhoods of a stable equilibrium. The moderate deviation scaling allows for a local approximation of the nonlinear dynamics by their linearized version and a finite dimensional subspace where exits take place with high probability is identified. Using stochastic control and variational methods we show that our scheme performs well both in the zero noise limit and pre-asymptotically. Simulation studies for stochastically perturbed bistable dynamics illustrate the theoretical results. This is joint work with Michael Salins and Konstantinos Spiliopoulos.

Pratima Hebbar (Duke University)
Limit theorems for branching diffusion processes

We describe the behavior of branching diffusion processes in periodic media. For a supercritical branching process, we distinguish two types of behavior for the normalized number of particles in a bounded domain, depending on the distance of the domain from the region where the bulk of the particles is located. At distances that grow linearly in time, we observe intermittency (i.e., the k-th moment dominates the k-th power of the first moment for some k), while, at distances that grow sub-linearly in time, we show that all the moments converge.

Jina Hyoungji Kim (Iowa State University)
From finite to infinite dimensions: functional inequalities for linear diffusions with degenerate noise
In this talk, we introduce functional inequalities for specific classes of hypoelliptic stochastic differential equations. In particular, these inequalities include the reverse log-Sobolev inequality and Wang-type Harnack inequality for a large class of linear SDEs with degenerate noise. They are obtained from gradient bounds of the semigroup of the process and making use of the generalized version of the carré du champ operator. From the results we take steps further and also look for quasi-invariance of the same form of SDEs that take values in an infinite-dimensional Hilbert space. This is to gain some type of smoothness of the heat kernel measure of an infinite-dimensional process, and done by using the "projected" bound.

Wenjian Liu (City University of New York)
Spectrum Analysis on a Positive Stochastic Process with Multiple Driving Processes
Bounded stochastic processes are becoming more intensely researched in recent years and has wide applications to many scientific areas. In this talk, we consider a general linear parabolic partial differential equation driven by a general multidimensional bounded stochastic process. Through in-depth spectrum analyses, we rigorously establish the uniqueness (up to positive scaling) of the solution to the parabolic PDE. We illustrate how to use the result by applying it to a general and important financial asset pricing problem. Specially, from observed option prices, the real-world probability distribution can be uniquely recovered, without scaling.

Liangbing Luo (University of Connecticut)
Logarithmic Sobolev inequalities on non-isotropic Heisenberg groups
We study logarithmic Sobolev inequalities with respect to a heat kernel measure on finite-dimensional and infinite-dimensional Heisenberg groups. Such a group is the simplest non-trivial example of a sub-Riemannian manifold. First we consider logarithmic Sobolev inequalities on non-isotropic Heisenberg groups. These inequalities are considered with respect to the hypoelliptic heat kernel measure, and we show that the logarithmic Sobolev constants can be chosen to be independent of the dimension of the underlying space. In this setting, a natural Laplacian is not an elliptic but a hypoelliptic operator. The argument relies on comparing logarithmic Sobolev constants for the threedimensional non-isotropic and isotropic Heisenberg groups, and tensorization of logarithmic Sobolev inequalities in the sub-Riemannian setting. Furthermore, these results can be applied in an infinite-dimensional setting to prove a logarithmic Sobolev inequality on an infinite-dimensional Heisenberg group modelled on an abstract Wiener space.

Vincent R Martinez (CUNY Hunter College)
On ergodicity of the damped-driven stochastically forced KdV equation
We discuss the existence, uniqueness, and regularity of invariant measures for damped-driven, stochastically forced KdV equation, where the noise is additive and sufficiently non-degenerate.

It is shown that a simple, but versatile control strategy, typically employed to establish exponential mixing for strongly dissipative systems such as the 2D Navier-Stokes equations, can nevertheless be applied in this weakly dissipative setting to establish elementary proofs of both unique ergodicity, albeit without mixing rates, as well as regularity of the support of the invariant measure. On the other hand, in the regime of large damping, we establish a one-force, one-solution principle, which ensures existence of a spectral gap with respect to a Wasserstein distance-like function.

Robert Neel (Lehigh University)
Localization and asymptotics of logarithmic derivatives of the heat kernel in small time
For a compact Riemannian manifold, bounds on the logarithmic derivatives of the heat kernel in small time have been known since the 90s, and generalizations to non-compact manifolds have been of recent interest. We show how stochastic methods naturally give the desired localization, even to incomplete Riemannian manifolds. Moreover, we give a probabilistic interpretation of the leading terms of the pointwise asymptotics, and we indicate the difficulties in the sub-Riemannian setting. This talk is based on joint work with Ludovic Sacchelli.

Dalton A R Sakthivadivel (Stony Brook University)
Statistical Inference and the Parallel Transport of Probability
Methods in statistics like maximum entropy usually focus on the probability measure associated to a dynamical system or field theory with probabilistic degrees of freedom; in so doing, statistical inference yields the probability of observing any given state of the system under some randomness. We prove that the solution to maximum entropy is parallel transport over the state space of the system. This gives a principled reason for the almost unreasonable success of certain energy-based statistical algorithms, in that finding the probability measure over states is simplified from solving a difficult PDE to an entirely geometric characterisation of a flow along a potential function.

Xiaoming Song (Drexel University)
Spatial averages for the Parabolic Anderson model driven by rough noise
In this paper, we study spatial averages for the parabolic Anderson model in the Skorohod sense driven by rough Gaussian noise, which is colored in space and time. We include the case of a fractional noise with Hurst parameters H_0 in time and H_1 in space, satisfying $H_0 \in (1/2, 1), H_1 \in (0, 1/2)$ and $H_0 + H_1 > 3/4$. Our main result is a functional central limit theorem for the spatial averages. As an important ingredient of our analysis, we present a Feynman-Kac formula that is new for these values of the Hurst parameters.

Jinwoo Sung (University of Chicago)
Minkowski content of Liouville quantum gravity metric spaces
A Liouville quantum gravity (LQG) surface is a "canonical" random two-dimensional surface, initially formulated as a random measure space and later as a random metric space. We show that the LQG metric almost surely determines the LQG measure, answering a question of Gwynne and Miller. More precisely, we prove that the LQG measure is almost surely a deterministic multiple of the Minkowski content measure for the LQG metric. Our primary tool is the continuum mating-of-trees theory for space-filling SLE. This is a joint work with Ewain Gwynne.

Jing Wang (Purdue University)
Spectral bounds for exit times of diffusions on metric measure spaces

We consider a diffusion on a metric measure space equipped with a local regular Dirichlet form. Assuming volume doubling property and heat kernel sub-Gaussian upper bound we obtain a spectral upper bound for the survival probability $\mathbb{P}_x(\tau_D > t)$ of the diffusion, where τ_D is its first exit time from domain D. Among other nice consequences, we are able to obtain a uniform upper bound for the product $\lambda(D) \sup_{x\in D} \mathbb{E}[\tau_D]$ where $\lambda(D)$ is the bottom of the spectrum. This is a joint work with Phanuel Mariano.

Kazuo Yamazaki (Texas Tech University)
Recent developments on probabilistic convex integration to prove non-uniqueness of stochastic PDEs
Convex integration is a technique that has its origin from differential geometry, specifically the work of Nash in 1954 concerning C1 isometric embedding theorem. Very recently, non-uniqueness in law of several stochastic partial differential equations have been proven via probabilistic convex integration; the list includes compressible or incompressible Euler equations, Navier-Stokes equations, Boussinesq system, and magnetohydrodynamics system. The types of noise include additive (white in time only or space-time white noise) linear multiplicative (in the interpretations of Ito or Stratonovich).