Union College Math Conference: Differential Geometry and Geometric Analysis

June 3–5, 2022

Friday program

4:30–5:30pm: Reception and registration (Olin Rotunda)

5:30–6:30pm: Plenary talk (Olin 115)

Tai Melcher: *Large deviations for sub-Riemannian random walks*

Saturday program

8:00–9:00am: Coffee & pastries, registration (Olin Rotunda)

9:00–10:30am: Session II (ISEC 220)

- 9:00–9:20 Ralph Gomez: *Themes of BHK Mirror Symmetry and Sasaki Manifolds*
- 9:30–9:50 Yury Ustinovskiy: *Real Monge-Ampere equations and toric Kähler-Ricci soliton*
- 10:00–10:20 Xumin Jiang: *Kähler-Einstein metrics on complex hyperbolic cusps*

10:30–11:00am: Coffee break (Olin Rotunda)

11:00am–12:00pm: Plenary talk (Olin 115)

Yusu Wang: *Weisfeiler-Lehman Meets Gromov-Wasserstein*

12:00–2:00pm: Lunch break

12:00–2:00pm: Women in Mathematics Lunch Panel (Bailey 207 - for people who preregistered for this event)

2:30–4:00pm: Session III (ISEC 220)

- 2:30–2:50 Ali Maalaoui: *Compactness of Einstein-Dirac structures*
- 3:00–3:20 Michael Albanese: *The Yamabe Invariant of Non-Kähler Surfaces*
- 3:30–3:50 Erin Griffin: *The Case for a General q-flow: An Investigation of Ambient Obstruction Solitons*

4:00–4:30pm: Coffee break with birthday cake (Olin Rotunda)

4:30–5:30pm: Plenary talk (Olin 115)

Claude LeBrun: *Four-Manifolds, Conformal Curvature, and Differential Topology*

6:00-8:30pm: Conference Dinner (Hale House)

Sunday program

8:00–8:30am: Coffee & pastries (Olin Rotunda)

8:30–11:00 am: Session IV (ISEC 220)

- 8:30–8:50 Spencer Whitehead: *Integrality theorems for symmetric instantons*
- 9:00–9:20 Elahe Khalili Samani: *Rational ellipticity of G-manifolds*
- 9:30–9:50 Joseph Hoisington: *Energy-Minimizing Mappings of Rank-*1 *Symmetric Spaces*
- 10:00–10:20 Brian Allen: *Sobolev Inequalities and Convergence of Riemannian Manifolds*
- 10:30-10:50 Ramesh Sharma: *Ricci and Yamabe Solitons In Contact Geometry*

11:00–11:30am: Coffee break (Olin Rotunda)

11:30am–12:30pm: Plenary talk (Olin 115)

Sergio Lopez-Permouth: *Collaborations among binary operations*

Abstracts

Claude LeBrun (Stony Brook)
Plenary talk: *Four-Manifolds, Conformal Curvature, and Differential Topology*
Gauss discovered that any Riemannian 2-manifold is locally conformally flat, in the sense that, near any point, there is a coordinate system in which the metric becomes a positive function times the Euclidean metric. However, this paradigm generally fails for Riemannian manifolds of higher dimension; in other words, most higher-dimensional manifolds are not "locally conformally flat". Indeed, when the dimension is at least 4, Weyl discovered that a piece of the Riemann curvature tensor, now known as the "Weyl tensor", vanishes identically if and only if the given metric is locally conformally flat.

Given a fixed smooth compact 4-manifold without boundary, the "Weyl functional" is by definition the L^2-norm-squared of the Weyl tensor, considered as a non-negative function on the space of Riemannian metrics. Its infimum over all metrics then provides a fascinating differential-topological invariant of the given smooth, compact 4-manifold. It turns out that there are many 4-manifolds for which this invariant can be exactly calculated, and there are even large classes of manifolds on which the infimum is achieved. However, our current understanding of this problem remains distinctly limited. In this talk, I will explain some recent results regarding this invariant, along with various conjectures that have guided some of my own forays into this still-mysterious territory.

Michael Albanese (UQAM)
The Yamabe Invariant of Non-Kähler Surfaces
The Yamabe invariant is a real-valued diffeomorphism invariant coming from Riemannian geometry. Using Seiberg-Witten theory, LeBrun showed that the sign of the Yamabe invariant of a Kähler surface is determined by its Kodaira dimension. We consider the extent to which this remains true when the Kähler hypothesis is removed. Partly based on joint work with Claude LeBrun.

Brian Allen (University of Hartford)
Sobolev Inequalities and Convergence of Riemannian Manifolds
Recently, it has been noticed that an analogue of Sobolev theory can be established between the norm of a Riemannian metric and its corresponding distance function. This observation leads to new compactness and convergence theorems and motivates many new questions. In this talk we will survey motivating conformal examples and then present several new results and how they are analogous to classical Sobolev inequalities.

Ralph Gomez (Swarthmore College)
Themes of BHK Mirror Symmetry and Sasaki Manifolds
The goal of this talk will be to use some ideas from a construction of mirror Calabi-Yau orbifolds due to Berglund, Hübsch, Krawitz (BHK mirror symmetry) to outline a way to create new examples of Sasaki manifolds of positive Ricci curvature. If time permits, we will say some things about the Sasaki-Einstein case. This is work in progress.

Erin Griffin (Seattle Pacific University)
The Case for a General q-flow: An Investigation of Ambient Obstruction Solitons
Consider a geometric flow by a general tensor, q, and the resulting solitons. We will see that we are able to draw a number of conclusions about solitons of the q-flow by prescribing various properties to q (e.g. being divergence-free).

Focusing on a specific q, we consider the ambient obstruction tensor, $\mathscr{O}$. We will see how our general results improve our understanding of ambient obstruction solitons and why this method is necessary for such an investigation.

Joseph Hoisington (University of Georgia)
*Energy-Minimizing Mappings of Rank-*1 *Symmetric Spaces*
We will prove sharp lower bounds for the energy of maps from real and complex projective spaces to Riemannian manifolds, and we will characterize the mappings which minimize energy in these results. These results lead to strong characterizations of energy-minimizing mappings of Kähler manifolds biholomorphic to complex projective space.

Xumin Jiang (Fordham University)
Kähler-Einstein metrics on complex hyperbolic cusps
Let L be a negative holomorphic line bundle over an $(n-1)$-dimensional complex torus D and let V be any closed tubular neighborhood of the zero section D in L. Datar-Fu-Song proved the existence of a unique complete Kähler-Einstein metric on V D if a boundary condition is assumed. We study the asymptotics of such KE metrics and prove a sharp exponentially decaying estimate. In addition, we study the asymptotics at 3 scales. This work is joint with Xin Fu and Hans-Joachim Hein.

Ali Maalaoui (Clark University)
Compactness of Einstein-Dirac structures
Let M be a compact manifold. We consider then the functional

$$E(g,\psi) = \int_M R_g + \langle D_g\psi, \rangle\psi - \lambda|\psi|^2 \, dv_g,$$

where g is a Riemannian metric and ψ is a spinor on M. The critical points of such a functional are called Einstein-Dirac structures. In this talk, I will discuss the compactness of sets of such structures. I will show that under fairly mild conditions the set of such structures is compact in dimension 3 but in dimension 4, there is compactness up to bubbling. The bubbling profile is also discussed.

Elahe Khalili Samani (University of Notre Dame)
Rational ellipticity of G-manifolds
Simply connected Riemannian manifolds with non-negative sectional curvature are conjectured to be rationally elliptic. While the conjecture is still open, it has been confirmed in several special cases. We generalize these results by removing the curvature assumption. Our result holds more generally in the context of singular Riemannian foliations. This is based on joint work with Marco Radeschi (University of Notre Dame).

Ramesh Sharma (University of New Haven)
Ricci and Yamabe Solitons In Contact Geometry
This talk will address the developments in the theory of Ricci solitons within the framework of contact geometry. In particular, it will indicate how a Heisenberg group (resp. solvable Lie group) are special cases of a Ricci soliton as a Sasakian (resp. (k, μ)-contact) metric. It will also provide some characterizations of a Ricci almost soliton (a generalization of a Ricci soliton) and Yamabe solitons in contact geometry.

Yury Ustinovskiy (Lehigh University)
Real Monge-Ampere equations and toric Kähler-Ricci solitons
Let (M^{2n}, ω) be a Kähler manifold equipped with a Hamiltonian action of a half-dimensional torus T^n. I will explain how the fundamental equations of the Kähler geometry (Kähler-Ricci flat, Kähler-Einstein and Ricci solitons) reduce to real Monge-Ampere equations for a convex function on the dual of the Lie algebra of the torus: $Lie(T^n)^*$. In a particular case of toric gradient steady Kähler-Ricci solitons I will give a rigidity result proving that the only complete solitons with a free T^n action are flat $(C^*)^n$.

Spencer Whitehead (University of Waterloo)
Integrality theorems for symmetric instantons
A symmetric instanton is a solution to the finite-energy anti-self-dual instanton equation on $\mathbb{R}^4$ which commutes with some perscribed group of symmetries. Symmetric instantons are classified by an analogue of the ADHM construction, in which maps are equivariant with respect to the symmetry group. This talk introduces the symmetric ADHM equations for structure group $\mathrm{SU}(N)$, and how index-theoretic methods can be used to derive integrality theorems for different symmetry groups. In the case of $N = 2$ and a discrete subgroup of $\mathrm{SU}(2)$, we derive the prime charge theorem, which imposes a canonical choice of representation for the equations at prime number charges, and show how it can be used to construct explicit instantons with the symmetries of a dodecahedron, icosahedron, icosidodecahedron, and truncated icosahedron.