Union College Math Conference: Rings and Algebras

June 3–5, 2022

Friday program

5:30–6:30pm: Plenary talk (Olin 115)
Tai Melcher: *Title TBA*

Saturday program

8:00–9:00am: Coffee & pastries, registration (Olin Rotunda)

9:00–10:30am: Session I (Bailey 201)

- 9:00–9:20 Hayden Julius: *Preserver problems concerning rank-one matrices*
- 9:30–9:50 Megan Chang-Lee: *TBA*
- 10:00–10:20 Ralph Morrison: *Higher distance commuting varieties*

10:30–11:00am: Coffee break (Olin Rotunda)

11:00am–12:00pm: Plenary talk (Olin 115)
Yusu Wang: *Title TBA*

12:00–2:00pm: Lunch break

2:00–3:00pm: Session II (Bailey 201)

- 2:00–2:20 Alexandre Tchernev: *Dynamical systems on chain complexes and canonical minimal resolutions*
- 2:30–2:50 Julianna Tymoczko: *The geometry and combinatorics of 3-row Springer fibers*

3:00–3:45pm: Featured talk (Bailey 207)
Kate Ponto: *Title TBA*

4:00–4:30pm: Coffee break (Olin Rotunda)

4:30–5:30pm: Plenary talk (Olin 115)
Claude LeBrun: *Title TBA*

Sunday program

8:00–9:00am: Coffee & pastries (Olin Rotunda)

9:00–10:30am: Session III (Bailey 201)

- 9:00–9:20 Jennifer Biermann: *Toric Ideals of Weighted Oriented Graphs*
- 9:30–9:50 Augustine O'Keefe: *Betti numbers of symmetric shifted ideals*
- 10:00–10:20 Anthony Iarrobino: *The mystery of commuting pairs of nilpotent matrices, and Jordan type*

10:30–11:30am: Coffee break (Olin Rotunda)

11:30am–12:30pm: Plenary talk (Olin 115)
Sergio López-Permouth: *Collaborations among binary operations*

12:30–1:45pm: Lunch break

Abstracts

Sergio López-Permouth: (Ohio University)
Plenary talk: *Collaborations among binary operations*
Given two binary operations, $*$ and $\circ$, on a set S, a third operation, $\square$ on S, is said to be a *collaboration between* $*$ *and* $\circ$ if, for all $a, b \in S$, $\square(a, b) \in \{*(a, b), \circ(a, b)\}$. Collaborations have also been named *two-option magmas* earlier, in order to emphasize their similarity with previously studied concepts such as *one-value magmas* and *two-value magmas*.

The dichotomy inherent to the definition of a collaboration makes it clear that one can use graphs to represent such operations. Take S to be the vertices and connect a with b when $\star$ is to be used (and not, otherwise). For that reason, the expression graph magmas has been associated to both one-value and two-value magmas.

Characterizations of associative one-value and two-value magmas are available in the literature. We ponder when a collaboration between two (not-necessarily associative) operations yield an associative operation. A lot of our discussion centers on the cases when $S = \mathbb{Z}$ and the operations $*$ and $\circ$ are addition and subtraction, and when $S = \mathbb{N}$ and the operations $*$ and $\circ$ are either addition and multiplication.

We report on an initial exploration of these concepts and will mention several problems that are suggested by them.

This talk is a report on a collaboration (no pun intended) with Majed Zailaee.

Kate Ponto: (University of Kentucky)
Featured talk: *TBA*
TBA

Jennifer Biermann (Hobart and William Smith Colleges)
Toric Ideals of Weighted Oriented Graphs
Toric ideals arise as the kernels of particular homomorphisms between polynomial rings. In this talk I will explain a construction for defining such a map from a finite simple graph and what we can learn about the structure of the ideal from the graph. I will then discuss what changes when you go from an unoriented graph to a vertex-weighted oriented graph.

Megan Chang-Lee (Brown University)
TBA
TBA

Anthony Iarrobino (Northeastern University)
The mystery of commuting pairs of nilpotent matrices, and Jordan type
What are the possible pairs of Jordan types for two commuting nilpotent matrices A, B? This is a striking mystery, with some clues. One of the clues is the pattern for the biggest Jordan type possible for B, given the Jordan type of A, when that for B has only two parts. The Jordan normal form for a map is named after Camille Jordan, who wrote about it in 1870: but this problem appears not to have been noticed by mathematicians until 2007 or so, when three different and independent groups wrote about it, including Polona Oblak with Tomaz Kosir from Ljubljana, Dmitri Panyushev from Moscow, and Leila Khatami, with Roberta Basili and I from Perugia, Schenectady and Boston. We suggest an analogy with the "curious incident of the dog in the night" in A. Conan Doyle's "Silver Blaze" of 1892 - no one heard the problem bark.

We report on past work with Leila Khatami, Bart Van Steirteghem, and Rui Zhao, and ongoing work with Leila, Bart, and Mats Boij; and we suggest this mystery of pairs of Jordan types for the attention of future detectives.

Hayden Julius (Youngstown State University)
Preserver problems concerning rank-one matrices
One of the cornerstones of the linear preserver problem (LPP) literature is the rank-one preserver problem, which asks for a description of linear maps acting on spaces of rectangular matrices preserving rank-one elements. Under mild technical conditions, these maps must be equivalence transformations (up to transposition, if the matrices are square); that is, a map of the form $X \mapsto UXV$, where U and V are fixed invertible matrices of appropriate sizes. The convenient description of rank-one preserving maps is directly related to the ordinary and Lie product structure of matrix rings and their homomorphisms. In this talk, we will formulate a new type of preserver problem concerning rank-one matrices, and in particular, rank-one idempotents and nilpotents, with surprisingly strong conclusions. We also present some open problems and pathologies that arise from these modified hypotheses.

Ralph Morrison (Williams College)
Higher distance commuting varieties
We say a pair of $n \times n$ matrices A and B commute if $AB = BA$. We can use this to define the commuting distance of two matrices, which measures the shortest chain of pairwise commuting nonscalar matrices connecting them. In this talk we prove that the set of pairs of matrices with

commuting distance bounded by any fixed d is defined by polynomial equations, at least over an algebraically closed field. This is joint work with Madeleine Elyze, Alexander Guterman, and Klemen Sivic.

Augustine O'Keefe (Connecticut College)
Betti numbers of symmetric shifted ideals
We introduce a new class of monomial ideals which we call symmetric shifted ideals. Symmetric shifted ideals are fixed by the natural action of the symmetric group and, within the class of monomial ideals fixed by this action, they can be considered as an analogue of stable monomial ideals within the class of monomial ideals. We show that a symmetric shifted ideal has linear quotients and compute its (equivariant) graded Betti numbers. As an application of this result, we obtain several consequences for graded Betti numbers of symbolic powers of defining ideals of star configurations.

Alexandre Tchernev (University at Albany - SUNY)
Dynamical systems on chain complexes and canonical minimal resolutions
We will discuss how to define in the very general context of chain complexes notions of "vector field" and its (time discrete) "flow". The resulting dynamical systems theory allows to construct in an explicit (and often canonical and symmetry-preserving) way from a given chain complex a "smaller" one of the same homotopy type. We will explain how to use this new theory to construct in an explicit, canonical, and symmetry- preserving way minimal free resolutions for every monomial and every toric ideal.

Julianna Tymoczko (Smith College)
The geometry and combinatorics of 3-row Springer fibers
The Springer fiber of a matrix X is a subvariety of the flag variety consisting of the flags fixed by X. One of the classical examples of geometric representation theory shows that their cohomology admits a representation of the symmetric group (or the Weyl group, in general Lie type). Like Schubert varieties, the geometry of Springer fibers is deeply entwined with combinatorics. Unlike Schubert varieties, very little is known about even straightforward questions about this geometry. In this talk, we study the combinatorics and geometry of a particular family of Springer fibers that arise in combinatorics, representation theory, and knot theory. We give some results about how to partition these Springer fibers into cells that are encoded by a kind of graph called a web.