Union College Math Conference: Featured Talks

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Abstracts

Claude LeBrun (Stony Brook University)
Four-Manifolds, Conformal Curvature, and Differential Topology

Gauss discovered that any Riemannian 2-manifold is locally conformally flat, in the sense that, near any point, there is a coordinate system in which the metric becomes a positive function times the Euclidean metric. However, this paradigm generally fails for Riemannian manifolds of higher dimension; in other words, most higher-dimensional manifolds are not "locally conformally flat". Indeed, when the dimension is at least 4, Weyl discovered that a piece of the Riemann curvature tensor, now known as the "Weyl tensor", vanishes identically if and only if the given metric is locally conformally flat. Given a fixed smooth compact 4-manifold without boundary, the "Weyl functional" is by definition the L 2 -norm-squared of the Weyl tensor, considered as a non-negative function on the space of Riemannian metrics. Its infimum over all metrics then provides a fascinating differential-topological invariant of the given smooth, compact 4-manifold. It turns out that there are many 4-manifolds for which this invariant can be exactly calculated, and there are even large classes of manifolds on which the infimum is achieved. However, our current understanding of this problem remains distinctly limited. In this talk, I will explain some recent results regarding this invariant, along with various conjectures that have guided some of my own forays into this still-mysterious territory

Sergio López-Permouth (Ohio University)
Collaborations among binary operations

Given two binary operations, $*$ and $\circ$, on a set S, a third operation, $\Box$ on S, is said to be a *collaboration between* $*$ *and* $\circ$ if, for all $a, b \in S$, $\Box(a, b) \in \{*(a, b), \circ(a, b)\}$. Collaborations have also been named *two-option magmas* earlier, in order to emphasize their similarity with previously studied concepts such as *one-value magmas* and *two-value magmas*.

The dichotomy inherent to the definition of a collaboration makes it clear that one can use graphs to represent such operations. Take S to be the vertices and connect a with b when $\star$ is to be used (and not, otherwise). For that reason, the expression graph magmas has been associated to both one-value and two-value magmas.

Characterizations of associative one-value and two-value magmas are available in the literature. We ponder when a collaboration between two (not-necessarily associative) operations yield an associative operation. A lot of our discussion centers on the cases when $S = \mathbb{Z}$ and the operations $*$ and $\circ$ are addition and subtraction, and when $S = \mathbb{N}$ and the operations $*$ and $\circ$ are either addition and multiplication.

We report on an initial exploration of these concepts and will mention several problems that are suggested by them.

This talk is a report on a collaboration (no pun intended) with Majed Zailaee.

Tai Melcher (University of Virginia)
Large deviations for sub-Riemannian random walks

One of the first important theorems one learns in probability is the law of large numbers, a primitive version of which says that, for S_n the number of heads in n tosses of a fair coin, the relative frequency of heads S_n/n converges to 1/2. The process S_n is an example of a random walk on R. A large deviations principle quantifies this convergence by finding the asymptotic rate of decay of the probability of events like S_n/n being larger than 1/2 for large values of n. More generally, large deviations techniques are used to study the exponential rate of decay of probabilities of increasingly unlikely events. These results have applications, for example, in statistical mechanics, information theory, and insurance.

In this talk we will discuss large deviations results for random walks on stratified nilpotent Lie groups. For such groups, there is a collection of vectors generating the Lie algebra, which equips the group with a natural but degenerate geometry. We consider random walks with increments in only these directions and show that, under certain constraints on the distribution of the increments, a large deviation principle holds with a natural rate function adapted to the subRiemannian geometry of these spaces.

This is joint work with Jing Wang and Masha Gordina.

Kate Ponto (University of Kentucky)
Euler characteristics in algebra and topology

There is a shockingly easy to prove theorem that is very useful for describing how Euler classes for modules interact with Morita invariance and how Euler characteristics for spaces behave for fibrations. I'll show you this really helpful theorem and explain how to apply it in these disparate settings.

Yusu Wang (University of California, San Diego)
Weisfeiler-Lehman Meets Gromov-Wasserstein

The Weisfeiler-Lehman (WL) test is a classical procedure for graph isomorphism testing. The WL test has also been widely used both for designing graph kernels and for analyzing graph neural networks. In this talk, I will describe the so-called Weisfeiler-Lehman (WL) distance we recently introduced, which is a new notion of distance between labeled measure Markov chains (LMMCs), of which labeled graphs are special cases. The WL distance extends the WL test (in the sense that the former is positive if and only if the WL test can distinguish the two involved graphs) while at the same time it is polynomial time computable. It is also more discriminating than the distance between graphs used for defining the Wasserstein Weisfeiler-Lehman graph kernel. Inspired by the structure of the WL distance we identify a neural network architecture on LMMCs which turns out to be universal w.r.t. continuous functions defined on the space of all LMMCs (which includes all graphs) endowed with the WL distance. Furthermore, the WL distance turns out to be stable w.r.t. a natural variant of the Gromov-Wasserstein (GW) distance for comparing metric Markov chains that we identify. Hence, the WL distance can also be construed as a polynomial time lower bound for the GW distance which is in general NP-hard to compute. This is joint work with Samantha Chen, Sunhyuk Lim, Facundo Memoli and Zhengchao Wan.