Analysis of the Doomsday Algorithm

By

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ABSTRACT

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The Doomsday Algorithm is an algorithm in which you can put in any date and find out what day of the week it fell on. This algorithm was first invented by Carroll but then Conway simplified it when he came up with the Doomsday rule. The Doomsday rule establishes a date that you can base all other dates off of to find what day it landed on much quicker. Before establishment of this rule, it took significantly longer to determine what day the date fell on. This paper looks at how both algorithms work and why the Doomsday rule makes this algorithm substantially easier to do mentally.

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1. INTRODUCTION

The Doomsday Algorithm was invented by John H. Conway, a professor of mathematics at Princeton. However, the algorithm that he used as the basis was invented nearly a century before by Charles Lutwidge Dodgson. However, Dodgson was better known by his pen name; Lewis Carroll. And while Carroll's algorithm caught the attention of John Conway, most people are far more familiar with Lewis Carroll as the author of *Alice's Adventures in Wonderland*[1] This algorithm is one where you put in any date from any year and it will tell you what day of the week it landed on. For example, you would input February 9th, 1676, and the algorithm tells you that date landed on a Wednesday. It is generally believed that Carroll originally came up with this algorithm because he was the first one to publish anything about it.

Carroll published his algorithm in 1887 in Nature volume 35 [1]; in this publication he states that his "average time for doing any such question is about 20 seconds, I have little doubt that a rapid computer would not need 15." Carroll was correct in this assumption, but it would not happen until 1973, when Conway improved upon Carroll's algorithm. Carroll was also slightly wrong with this guess; it would not take Conway fifteen seconds to do his algorithm, but only about one to two seconds. Conway was able to drastically simplify the algorithm with the introduction of the Doomsday rule, hence the name "Doomsday Algorithm." With this rule, Conway was able to do this algorithm quicker than ever expected. Of course, you have to take into account

[1]Interestingly, Carroll's mathematician background seeps into Alice's Adventures in Wonderland, particularly at the Mad Hatter's tea party. That scene can be read as a critique of William Hamilton's work in quaternions. Hamilton noted the absence of time in his equation meant his calculations went round and round. The character Time did not attend the Mad Hatter's party and the characters who did attend simply go round and round.

that Conway is a genius, but for an ordinary person it might take anywhere from three to five seconds to do it in their head.

In this paper, we will look at how both Carroll's and Conway's algorithms work. It will explore how they were both able to accomplish the same goal with somewhat different methods. More importantly though, the paper will discuss how Conway drastically improved upon Carroll's method with the use of the Doomsday rule. The Doomsday rule took what was a somewhat complex calculation and simplified to the point where anyone with basic math skills can take a date and determine on what day of the week it falls. Of course, even now there are mathematicians and others trying to make this calculation even easier to do mentally. However, one must consider that it took eighty-six years before Conway improved upon Carrolls' method. Assuming the next improvement takes as long, we can expect a new algorithm sometime around January 1, 2059, which, using the Doomsday algorithm, would be a Saturday.

1.1. **MODULAR ARITHMETIC.** Johann Carl Friedrich Gauss is usually attributed with the invention/discovery of modular arithmetic. In 1796 he did some work that advanced the field, and in 1801 published the book *Disquisitiones Arithmeticae* which, amongst other things, introduced modular arithmetic and the $\equiv$ symbol. So he is the person that laid out the modern approach to modular arithmetic that we use today.

So in pure mathematical terms, modular arithmetic is defined as if $n > 1$ is an integer, and if a, b are integers, then we say a is congruent to b modulo n if n divides (a-b). By the definition of division, this is equivalent to saying that there exists an integer k such that $a = nk + b$, which is where you get the "remainder". We write out this as

$$a \equiv b \bmod n$$

The reason that this is important is because, since each week is 7 days long, we will work with mod 7 for all of the math. With this in mind, this means that each day must be given a value from 0 to 6. So therefore,

- Sunday = 0
- Monday = 1
- Tuesday = 2
- Wednesday = 3
- Thursday = 4
- Friday = 5
- Saturday = 6

It is important to remember this because when doing the modular arithmetic the number you get corresponds to the day that the date falls on.

1.2. **RULES OF THE CALENDAR.** Calendars have actually changed quite a bit since their conception. First there was the Egyptian calendar of exactly 365 days, but that was changed by Julius Caesar. Caesar changed the calendar to $365\frac{1}{4}$ days to better reflect the true length of the year. This meant that every 4 years would be a leap year. However, it was discovered the actual length of the year is even closer to 365.242 days. Although this may not seem like a huge difference, this difference adds up over hundreds and hundreds of years. So much so that when the calendar was changed again in 1582 by Pope Gregory, that 10 days had to be added.

This meant, for instance, that October 5, 1582 became October 15, 1582. However, this change was not made uniformly throughout the entire world. Different countries changed to the Gregorian Calendar in different years. For example: Italy, Spain, and France changed over in the year above; but Britain and the American colonies did not change over until September 3, 1752. Of course countries all over the world also changed at other times with the latest being Greece in 1923. For the sake of this algorithm though, if somebody were to ask you about October 8, 1582 you would treat it as though it was a date on the Gregorian calendar.

The last thing to be noted about calendars is that there are some years that are divisible by 4 but are not leap years. These years are ones that are divisible by 100, unless it is divisible by 400. So these years are not leap years: 1500, 1700, 1800, and 1900, but 1600 and 2000 are leap years.

2. DOOMSDAY ALGORITHM

For decades, John Conway has been a man obsessed with games. "Conway likes games that move in a flash. He played backgammon constantly, for small stakes—money, chalk, honor—though for all that practice he was not terribly good at backgammon, either" (69).[7] Despite being an avid backgammon player, Conway still found time for all of his research. Conway discovered many things. The one he was best known for though, was his cellular automaton called the Game of Life, or just Life. One of Conway's many other discoveries was the creation of the Doomsday Algorithm.

Conway was so enthralled with Carroll's algorithm that he decided to make it better. He did so by creating the "Doomsday Rule." Conway became incredibly proficient at it being able to do this algorithm in his head, taking only about one second to complete the task. This is possible because the algorithm has a bunch of rules and tricks to make it possible to do the algorithm in your head. All of these rules and tricks are summed up in this poem [6]:

> "The last of Feb., or of Jan. will do
> (Except that in Leap Years it's Jan. 32)
> Then for even months use the month's own day,
> And for odd ones add 4, or take it away
>
> Now to work out your Doomsday the orthodox way
> Three things you should add to the century day
> Dozens, remainders, and fours in the latter,
> (If you alter by sevens of course it won't matter)

In Julian times, lackaday, lackaday
Zero was Sunday, centuries fell back a day
But Gregorian 4 hundreds are always Tues.
And now centuries extra take us back twos.
According to length or simply remember,
you only subtract for September, or November."

Although Conway invented the Doomsday Algorithm, there was an algorithm made 100 years before him that performed the same function. This algorithm though, was quite a bit more complex because it did not use the "Doomsday Rule."

The Doomsday Rule is actually quite a simple rule. Conway defines a Doomsday as the day of the week in which the last day of February falls on in any given year. The reason for this is because February is the only month that adds a day in certain years. From here, Conway noticed that no matter what year it is or what calendar is being used that certain dates in the calendar always fell on the same day.

These dates are: January 3rd (4th), February 28th (29th), March 7th, April 4th, May 9th, June 6th, July 11th, August 8th, September 5th, October 10th, November 7th, December 12th. The parentheses show that during leap years the Doomsdays for February and January are different, than from any other given year. Some examples of these are as follows:

- In 1793 the Doomsday was a Thursday
- In 1681 the Doomsday was a Friday
- In 1812, a leap year, the Doomsday was a Saturday
- Then again in 1253, the Doomsday was a Friday

Conway chose these days as his Doomsday in each month because they are relatively easy to remember. When you look at these days listed out numerically you can see that there is a trick to these dates. First, take a look at all the even months, besides February: 4/4, 6/6, 8/8, 10/10, and 12/12. One can see that the even dates are incredibly easy to memorize as their Doomsday is the same number as the month of the date itself. The odd months however, also have a somewhat easy rule, although not quite as easy as the even months. For the odd months with 31 days you add four to the number of the month. So it is: 3/7, 5/9, and 7/11. But for the odd months with only 30 days you subtract four from the number of the month, so it goes 9/4 and 11/7. The one exception is January, which has 31 days but you do not add 4 to the month.

Month:	January	February	March	April	May	June	July	August	September	October	November	December
Doomsday:	1/3(4)	2/28(29)	3/7	4/4	5/9	6/6	7/11	8/8	9/4	10/10	11/7	12/12

Table 1. Doomsdays

Of course these do not have to be your Doomsdays if you do not want, these were simply picked because of how easy they are to remember. You can even have multiple Doomsdays in a month if you please. The whole point of the Doomsday is to have certain days in the month off of which you base all other days. So in July for example if 7/11 does not work for you, you could always make 7/4, Independence Day, your Doomsday. Examples of this are:

- In 1253 if you want July 4th to be your Doomsday instead of July 11th, it would still be a Friday
- Or if you rather prefer Boxing Day, December 26th, 1110 then your Doomsday would be Monday

- Or take 1935 when the Doomsday is Thursday, then 11/28 which is Thanksgiving this year could be your Doomsday

This can be done with every other month as well if there is a certain holiday that makes that month easier to remember for you. The whole point of the Doomsday rule is to turn finding the day into simple modular arithmetic. Although, finding the day is now just simple math at this point, this can still be confusing because we are adding numbers to days. So Conway came up with a mnemonic for this problem as well; it goes like this:

NUN-day	ONE-day	TWOS-day	TREBLES-day	FOURS-day	FIVE-day	SIXER-day	SE'EN-day
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday

Table 2. A table of mnemonic devices for days of the week

The best way to explain how this rule works is through examples:

- Take January 1st, 400. The Doomsday in 400 is a Wednesday/TREBLES-day. So take the closest Doomsday to the desired date. In this case January 4th, and 1/1 is 3 days behind it so, $3 - 3 = 0$. So 1/1 is NUN-day or Sunday.
- Say you want to know May 28th, 1500. Then we take the Doomsday of 5/23 which is a Wednesday or TREBLES-day and then add 5, since the 28th is 5 days after the 23rd. So $3 + 5 = 8 \equiv 1 \bmod 7$. So 5/28 is ONE-day or Monday
- April Fools in 1702; the Doomsday is Saturday/SIXER-day. So 4/4 lies on Saturday and as April Fools is 4/1 we must subtract 3, thus $6-3 = 3$. Hence, April 1st, or April Fools, in 1702 falls on a TREBLES-day or Wednesday

So with this rule in mind, the hard part becomes finding what day the Doomsday lies on in that specific year. This though also has a rule to make it possible to do it in your head as well. This rule goes like this; you take the Doomsday from the beginning of the century add then add [4]:

> The number of DOZENS after that year,
> The REMAINDER after this, and
> The number of FOURS in the remainder.

It is interesting to note that in *Winning Ways* this equation is never explicitly stated, but one can find from other sources that this is the general equation to finding Doomsdays [2]:

$$\text{Doomsday from century year} + \frac{x}{12} + x\text{mod}12 + \frac{x\text{mod}12}{4}$$

where x is the last 2 digits of the desired year.

So in other words, that equation is the numerical version of the poem listed above. This is helpful if one wanted to memorize an equation instead of memorizing the poem.

Examples of this rule are as follows:

- Take 1470, the Doomsday in 1400 is Sunday aka NUN-day; so $0 + 5(dozen) + 8(remainder) + 2(fours) = 17 \equiv 3 \text{ mod} 7$. So the Doomsday in 1470 would be TREBLES-day, or Wednesday
- Take 2327, the Doomsday in 2300 is Wednesday aka TREBLES-day; so $3 + 2(dozen) + 3(remainder) + 0(fours) = 8 \equiv 1 \text{ mod} 7$. So the Doomsday in 2327 is ONE-day or Monday

There is some memorization that goes into this part though. Not only do you have to remember the rule above, but you also have to remember the base Doomsday for the century. Luckily though, Conway made a trick to remember this as well. For the Julian calendar, which stops at the end of the 17th century, the rule goes like this: Each century is one day earlier than the previous [6]:

Sun	Mon	Tues	Wed	Thurs	Fri	Sat
0						
		...	400	300	200	100
					1600	1500

Table 3. Rule for Julian calendar

For the Gregorian Calendar the rule is slightly different. The rule for this calendar is two days earlier than the previous century [6]:

Sunday	Tuesday	Wednesday	Friday
1700	1600	1500	
2100	2000	1900	1800
		...	2200

Table 4. Rule for Gregorian calendar

It is interesting to note what happens with Doomsdays every 12 years. It is called the twelve-year rule, even though it is not a rule per say. This is simply a fact that says every 12-year jump forward moves the Doomsday ahead by one day, this is true for both leap and non-leap years. The reason for this is because moving forward 12 years moves the Doomsday by 15 days. This is because every 12-year jump forward (up to the 99th year in a century) contains 3 leap years and 9 non-leap years; and $3 * 2 + 9 * 1 = 15$. And as $15 \equiv 1 \mod 7$, hence, the Doomsday moves by one day in 12 year jumps.

The last part of finding the Doomsday has to do with the B.C.E years. This as well is also a pretty easy rule to follow and to do in your head. This is not really an additional rule, it is more of a trick, really. You simply add a multiple of 28, or 700, big enough to turn it into a C.E. year. Then from here you just find the Doomsday by using the rules for the Julian or Gregorian Calendar depending on the year that you added to. For example, take the date September 1st, 3004 B.C.E then add 3500 to it. Giving us September 1st, 457 C.E which would be a Sunday.

Here are other examples of finding various dates using the Doomsday algorithm:

- March 20th, 533 is a Sunday. The original Doomsday of March 7th, 533 was a Monday
- October 31st, 1536 is a Tuesday. The original Doomsday of October 10th, 1536 was a Tuesday
- November 1st, 181 is a Wednesday. The original Doomsday of November 7th, 181 was a Tuesday
- February 22nd, 2222 is a Friday. The original Doomsday of February 2nd, 2222 is a Thursday
- April 20th, 2530 is a Thursday. The original Doomsday of April 4th, 2530 is a Tuesday

To do all of these examples takes less than 3 minutes once you have learned all the rules thoroughly.

3. CARROLL'S FORMULATION

Lewis Carroll was the first person to publish an algorithm to find the date for any given day, therefore he is credited with the creation of this algorithm. This was the algorithm that Conway would later use to build his Doomsday rule. So naturally, the algorithm created by Carroll has similarities to that of Conway's algorithm, discussed in the previous chapter. For now, the main difference to be considered between the two is the use of the Doomsday rule. While this chapter will discuss the differences in more detail, we must first understand the basis of Carroll's equation.

Carroll's equation was also able to be done mentally in quite a short amount of time; Carroll claimed in his article that he could complete the calculations in approximately 20 seconds. He was positive though, that someone or something else could do this much faster than he could. This equation also uses modular arithmetic to figure out what day any given date will fall on, although there are far fewer mnemonics to help one remember.

This is where his formula gets a bit weird, though. So, to explain his findings thoroughly we will follow an example from the text closely. [5] Recall that prior to 1582, the Julian calendar was used. Beginning in October of 1582, there was a switch to the Gregorian calendar, which is still in use today. To account for the change over from the Julian calendar to the Gregorian calendar, the first month of the year is March and not January like we are accustomed to. So for instance, February would be the 12th month while June is the 4th month. So let,

- k = day of the month
- m = month,

with

- March = 1
- April = 2
- May = 3
- June = 4
- July = 5
- August = 6
- September = 7
- October = 8
- November = 9
- December = 10
- January = 11
- February = 12
- N = year

One thing to keep in mind is that with the months set up like this, February 1875 is the 12th month of 1874 while April, 1875 would be the second month of 1875. So with this, N is the current year as long as the month is not January or February. So to find the year use the equation N = 100C + Y where

- C = Century
- Y = particular year of the century

With all of this, one is almost able to deduce the equation.

As March 1st is the basis for Carroll's equation we will use March 1st, 1600 to show how the equation comes about. So let d_N represent the day of the week and let N be the year. Then notice that $365 \equiv 1 \text{ mod} 7$, so $d_N \equiv d_{N-1} + 1 \text{ mod} 7$.

But if N is the leap year, there is an extra day, so the equation becomes $d_N \equiv d_{N-1} + 2 \bmod 7$.

Hence, to find how many leap years have passed between d_N and d_{1600}, lets call this x. So to find this we must now how many years are divisible by 4, 100, and 400. Therefore, the equation we use is

x= [(N-1600)/4] - [(N-1600)/100] + [(N-1600)/400]

x= [N/4] - [N/100] + [N/400] - 388

Now lets put this in terms of C and Y. After some simplifying, we obtain

$x \equiv 3C + [C/4] + [Y/4] - 3 \pmod 7$

When doing [C/4] and [Y/4] that one takes however many fours are in said number. Take the year 1595 for example. Then [15/4]= 3 while [95/4]=23.

One can now compute d_N from d_{1600} by shifting d_{1600} by one day for every year that has passed, plus an extra days for leap years. Thus

$d_N \equiv d_{1600} + N + x$ [Simplifying this we get,]

$d_N \equiv d_{1600} - 2C + Y + [C/4] + [Y/4] \pmod 7$

So as $d_{1600} = 3$, so March 1st, 1600 is a Wednesday. So as Wednesday = 3 the formula becomes [5]:

$$D_N \equiv 3 - 2C + Y + (C/4) + (Y/4) \pmod 7$$

Now we must look at the change in days from month to month. When we look at the change from March 1st to April 1st we see that there are 31 days in between, hence, $31 \equiv 3 \bmod 7$. Likewise, from April 1st to May 1st it is $30 \equiv 2 \bmod 7$. Thus for all the months the difference is either 2 or 3 depending on how many days are in the month. So

- From March 1 to April 1 = 3 days

- April 1 to May 1 = 2 days
- ...
- December 1 to January 1 = 3 days
- January 1 to February 1 = 3 days

So now, we must come up with a formula that gives us the same increments. This formula was actually founded by Reverend Zeller [5], through trial and error. This formula is as follows $d_N + [2.6m - 0.2] - 2$. When doing $[2.6m - 0.2]$ one rounds to the closest whole number.

So with this we are able to come up with the final equation. We simply add k-1 to the equation and we get

$$W \equiv k + [2.6m - 0.2] - 2C + Y + [Y/4] + [C/4] \pmod{7}$$

where W = day of the week, k=the given date, and m= month

Take the date August 9th, 1681. When we plug this into the equation we get $W \equiv 9 + [2.6(6) - 0.2] - 2(16) + 81 + [81/4] + [16/4](\text{mod}7)$ Which after doing this mentally you get $W \equiv 2 \bmod 7$. So August 9th, 1681 is a Tuesday.

3.1. **SIMILARITIES AND DIFFERENCES.** Although Conway's algorithm may have been based off of Carroll's algorithm, one can clearly see that Conway changed it quite a bit. The equation for Carroll's algorithm is actually quite complicated and, by today's standards, would be quite hard to do mentally. Luckily though, Conway substantially simplified it. Not only did he simplify it, but he came up with different rules and mnemonics that would make it even easier for people nowadays to mentally work through the steps. Conway's algorithm is actually so easy to do that one can do it with just one hand.

Conway spent weeks trying to revise Carroll's algorithm into the Doomsday algorithm that we know today. Obviously, the algorithms both find the day that any date falls on. Conway's equation (1) requires the use of fewer inputs but uses two distinct modulo, (2) is much easier to do mentally, and (3) permits the use of mnemonics to speed up the calculations.

However, it is Conway's use of Doomsdays that creates the greatest difference between the two algorithms. As discussed in the *DOOMSDAY ALGORITHM* section above, the equation for finding the Doomsday for any given year is as follows:

$$\text{Doomsday from century year} + \frac{x}{12} + x\text{mod}12 + \frac{x\text{mod}12}{4}$$

where x is the last 2 digits of the desired year

But to make this comparison fair, we must change Conway's equation so that it also computes the date in one step. This equation would look like:

$$\text{Doomsday from century year} + \frac{x}{12} + x\text{mod}12 + \frac{x\text{mod}12}{4} + d \mod 7$$

where x is the same as above, and

d = difference between the Doomsday and the desired date.

An example of using Conway's equation is as follows:

- Take May 28th, 1470 and plug it into the previous equation. Note that the Doomsday in 1400 is Sunday or NUN-day and 28 is 19 days from 5/9 (the Doomsday in May), plugging into the equation we obtain:

  $0 + \frac{70}{12} + 70 \mod 12 + \frac{70\text{mod}12}{4} + 19 \mod 7$ [Simplifying gives us]

  $0 + 5 + 8 + 2 + 19 = 34 \equiv 6 \mod 7$ Thus May, 28th, 1470 is a Saturday

Carroll's equation, on the other hand, is:

$$W \equiv k + [2.6m - 0.2] - 2C + Y + [Y/4] + [C/4]\ (\text{mod}7)$$

When plugging in the same date to Carroll's equation we get:

$W \equiv 28 + [2.6(3) - 0.2] - 2(14) + 70 + [70/4] + [14/4](\mod 7)$ [simplifying]

$W \equiv 28 + 7 - 28 + 70 + 17 + 3 = 97 \equiv 6 \mod 7$ So once again, we see that it is a Saturday.

With the two equations side by side, it is quite easy to see just how drastically Conway changed the original equation. Carroll's equation has 4 separate inputs, compared to Conway's which has only 2 inputs to worry about. As mentioned before though, the downside to this is that Conway's equation uses

2 different modulos ;you have to use both mod 12 and mod 7. But this actually makes Conway's equation easier for one to do mentally.

The reason why it is so much easier to do Conway's equation mentally than that of Carroll's is because of the use mod 12. Both equations use mod 7, but only Conway's equation uses mod 12. This is because Conway noticed the 12-year rule, which states that Doomsday moves ahead exactly one day every 12-year leap forward. Because of this finding, Conway is able to introduce mod 12 into his equation. This means that the biggest number that someone may have to add in Conway's formula is 11. Meanwhile in Carroll's formula someone could add 99 if Y=99, which by today's standards is quite hard to do mentally. When looking at the example used for Conway's equation, one may notice that d=19, which is clearly bigger to 11. This occurred only because I kept the Doomsday as May, 9th. But, if I used the Doomsday of May, 23rd then $d = 5$, which is clearly smaller than 12. This is one of the many tips and tricks that Conway introduces to make his equation much easier to use.

The other clear difference between the two equations is that one has to memorize more when trying to use Conway's formula. Not only does one have to memorize the equation for Conway's but they also have to memorize the Doomsdays of each century. Conway though makes this much easier then it initially sounds, as discussed in greater detail in Chapter 2, because he came up with tricks as to make this memorization easier. From finding the Doomsday of a specific year down to making clever names for the days of the week, Conway has a trick or tip to make his algorithm all the much easier. In contrast, Carroll simply just shows one how to derive the formula for finding the day of any given date.

At first glance, Carroll's one step formula might seem easier than Conway's two step process with his Doomsday rule. However, because Conway developed mnemonics and other memory aids, his process represents an improvement on Carroll's equation. Plus, it is relatively easy for one to turn the equation for finding the Doomsday into an equation that finds any given date.

3.2. **ROOM FOR IMPROVEMENT.** There have been some suggestions to making finding the Doomsday easier. There have not been any substantial improvements to Conway's algorithm as of today. There has only been one proposed change to Conway's algorithm. That change is to change the equation to find the Doomsday of any given year, which looks like:

$$\frac{x}{12} + x\text{mod}12 + \frac{x\text{mod}12}{4}$$

where x is the last two digits of the desired year.

To this equation [3]:

$$2y + 10(y\text{mod}2) + z + \frac{2(y\text{mod}2 + z)}{4}$$

where y is the tens digit of x, and z is the ones digit of x, i.e $x = 10y + z$

As one can see this is not a gigantic improvement, especially when you think about the change from Carroll's equation to Conway's. The whole point of this change is to make it easier to find the Doomsday mentally. It does this by only requiring 3 additions and a multiplication by 2. And it has been broken down into single digits, by separating the tens and ones digits, so this will also make it easier to do mentally.

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