Union College Math Conference: Differential Geometry and Geometric Analysis

September 13–15, 2019

Friday program

4:30–5:30pm: Reception and registration (Olin Rotunda)

5:30–6:30pm: Plenary talk (Olin 115)
John McCleary: *The Oberwolfach archives and topology*

Saturday program

8:00–9:00am: Coffee & pastries, registration (Olin Rotunda)

9:00–10:30am: Session I (Lippman 012)

- 9:00–9:25 Lee Kennard: *Positive curvature and torus symmetry*
- 9:30–9:55 Demetre Kazaras: *Desingularizing positive scalar curvature metrics on 4-manifolds*
- 10:00–10:25 Frank Morgan: *Double bubbles in space with density*

10:30–11:00am: Coffee break (Olin Rotunda)

11:00am–12:00pm: Plenary talk (Olin 115)
Álvaro Lozano-Robledo: *Recent progress on the classification of torsion subgroups of elliptic curves*

12:00–2:00pm: Lunch break

2:00–4:00pm: Session IIa (Lippman 012)

- 2:00–2:25 Alice Lim: *Loops to infinity and beyond*
- 2:30–2:55 Daniel Martin: *Rigidity of positive mass theorems in general relativity*
- 3:00–3:25 Brian Allen: *Properties of the null distance on a spacetime*
- 3:30–3:55 Carlos Vega: *Splitting spacetime*

2:00–4:00pm: Session IIb (Lippman 016)

- 2:00–2:25 Scott Wilson: *A generalization of the Kähler symmetries to Hermitian and almost-Kähler manifolds*

- 2:30–2:55 Surena Hozoori: *Ricci-Reeb realization problem and contact 3-manifolds*
- 3:00–3:25 Marlon Gomes: *Conics, twistors, and anti-self-dual metrics*
- 3:30–3:55 Michael Albanese: *The Yamabe invariant of Inoue surfaces*

4:00–4:30pm: Coffee break (Olin Rotunda)

4:30–5:30pm: Plenary talk (Olin 115)
Robert Ghrist: *Applied Topology: Sheaves and Cosheaves*

6:00–8:30 pm: Conference Dinner (Old Chapel)

Sunday program

8:00–8:30am: Coffee & pastries (Olin Rotunda)

8:30–10:30am: Session III (Lippman 012)

- 8:30–8:55 Robert Ream: *Weyl-minimal surfaces and the adjunction inequality*
- 9:00–9:25 Gideon Maschler: *Kähler-Einstein metrics and shear*
- 9:30–9:55 Amir Babak Aazami: *Distinguished Kähler 4-manifolds via Lorentzian geometry*
- 10:00–10:25 William Wylie: *Restrictions on topology of extreme black holes*

10:30–11:00am: Coffee break (Olin Rotunda)

11:00am–12:00pm: Plenary talk (Olin 115)
Carolyn Gordon: *Decoding geometry and topology from the Steklov spectrum of orbisurfaces*

12:00–1:45pm: Lunch break

1:45–3:45pm: Session IV (Lippman 012)

- 1:45–2:10 Ivan Contreras: *Split canonical relations and geometric quantization*
- 2:15–2:40 Florin Catrina: *On the hypercontractivity of a convolution operator*
- 2:45–3:10 Ram Shankar Gupta: *Lightlike submanifolds of an indefinite almost contact metric manifold*
- 3:15–3:40 Arifa Mirza: *Bilinearization and soliton solutions of $N = 1$ supersymmetric coupled dispersionless integrable system*

Abstracts

Carolyn Gordon (Dartmouth College)
Plenary talk: *Decoding geometry and topology from the Steklov spectrum of orbisurfaces*
The Dirichlet-to-Neumann or "voltage-to-current" operator of, say, a surface M with boundary is a linear map $C^\infty(\partial M) \to C^\infty(\partial M)$ that maps the Dirichlet boundary values of each harmonic function f on M to the Neumann boundary values of f. The spectrum of this operator is discrete and is called the Steklov spectrum. The Dirichlet-to-Neumann operator also generalizes to the setting of orbifolds, e.g., cones. We will address the extent to which the Steklov spectrum encodes the geometry and topology of the surface or orbifold and, in particular, whether it recognizes the presence of orbifold singularities such as cone points.

This is joint work with Teresa Arias-Marco, Emily Dryden, Asma Hassannezhad, Elizabeth Stanhope and Allie Ray.

Amir Babak Aazami (Clark University)
Distinguished Kähler 4-manifolds via Lorentzian geometry
Given a Lorentzian 4-manifold (M, g) with two distinguished vector fields satisfying properties determined by their shear, twist and various Lie bracket relations, a family of Kähler metrics g_K is constructed on M. Under certain conditions g and g_K share various properties, such as a Killing vector field or a vector field with geodesic flow. The Ricci and scalar curvatures of g_K are computed in some cases in terms of data associated to g; in certain cases the Kähler manifold (M, g_K) will be complete and Einstein. Many classical spacetimes fit into this construction: warped products, for instance de Sitter spacetime, as well as gravitational plane waves and metrics of Petrov type D, such as Kerr and NUT metrics. This work is joint with Gideon Maschler.

Michael Albanese (UQAM)
The Yamabe invariant of Inoue surfaces
The Yamabe invariant is a real-valued diffeomorphism invariant coming from Riemannian geometry. Using Seiberg-Witten theory, LeBrun showed that the sign of the Yamabe invariant of a Kähler surface is determined by its Kodaira dimension. We show that the Yamabe invariant of Inoue surfaces and their blowups is zero which demonstrates that the non-Kähler analogoue of LeBrun's theorem does not hold.

Brian Allen (University of Hartford)
Properties of the null distance on a spacetime
The null distance was introduced by Christina Sormani and Carlos Vega as a way of turning a spacetime into a metric space. This is particularly important for geometric stability questions relating to spacetimes. In this talk we will describe the null distance and present new properties of the metric space structure and its relation to the causal structure of a spacetime. This is joint work with Annegret Burtscher.

Florin Catrina (St. John's University)
On the hypercontractivity of a convolution operator
We discuss a convolution operator which appears as an integral representation of the Wick product on $L^p(\mathbb{R}, \mu)$ spaces where the probability measure μ has a Gamma distribution. For certain values of the parameters, the hypercontractivity of this operator is tightly connected to inequalities of Brascamp-Lieb type.

Ivan Contreras (Amherst College)
Split canonical relations and geometric quantization
Following Alan Weinstein's creed "everything is a Lagrangian submanifold", we develop the theory of split canonical relations, which appears naturally in the Hamiltonian formulation of field theory. We prove that the evolution relations of the Poisson sigma model are split, providing a new interpretation of the reduced phase space of the model. We explain the role of these relations in Hawkin's C*-algebra quantization program. Based on joint work with Alberto Cattaneo (https://arxiv.org/abs/1811.10107).

Marlon Gomes (Stony Brook University)
Conics, twistors, and anti-self-dual metrics
Penroses twistor space construction associates to a Riemannian, oriented 4-manifold M a real, 6-dimensional manifold Z, a fibration over M, whose fibers are 2-spheres, admitting a preferred almost-complex structure. This almost-complex structure is integrable if the Weyl tensor of the underlying Riemannian 4-manifold is anti-self-dual, that is, its Hodge-dual is its negative. Anti-self-dual manifolds are amenable to complex analysis, even if they possess no complex structure of their own, by means of their twistor spaces. The anti-canonical bundle of a twistor space admits a square-root. If this square-root has at least three independent sections, its linear system defines a holomorphic map to CP^2. The fibers of the twistor fibration project to conics under this map. The collection of conics in the image of this map spans a hypersurface in the space of conics in CP^2, the vanishing locus of a function. Functions arising from this construction satisfy a differential equation, linked to the conformal geometry of the space of conics. The function corresponding to the round metric on S^4 was described in simple form by Dunajski and Tod (2018). We present a detailed analysis of the Fubini-Study metric from this viewpoint, and describe an ansatz to produce anti-self-dual metrics explicitly by superposition of these two models.

Ram Shankar Gupta (Guru Gobind Singh Indraprastha University)
Lightlike submanifolds of an indefinite almost contact metric manifold
In the theory of submanifolds of semi-Riemannian manifolds it is interesting to study the geometry of lightlike submanifolds as the intersection of normal vector bundle and the tangent bundle is non-trivial. The geometry of lightlike submanifolds is used in mathematical physics, in particular, in general Relativity since lightlike submanifolds can be models of different types of horizons(event horizons, Cauchy's horizons, Kruskal's horizons).

In this talk, I will present some of my recent work on lightlike submanifolds of an indefinite almost contact metric manifold.

Surena Hozoori (Georgia Institute of Technology)
Ricci-Reeb realization problem and contact 3-manifolds
It is well known that in Riemannian geometry, local information can lead to global phenomena. On the other hand, in the category of contact manifolds, we can naturally focus on so called "compatible" Riemannian structures. However, it is very little known about how to use compatible global geometry to achieve contact topological information. We will discuss the problem of Ricci curvature realization for Reeb vector fields associated to a contact 3-manifold. These vector fields have significantly helped understanding contact topology since early 90s. We will use topological tools, namely open book decompositions, to show that every admissible function can be realized as such Ricci curvature for a singular metric which is an honest compatible metric away from a measure zero codimension one set. However, we will see

that resolving such singularities depends on contact topological data and is yet to be fully understood.

Demetre Kazaras (Stony Brook University)
Desingularizing positive scalar curvature metrics on 4-manifolds
We study 4-manifolds of positive scalar curvature (psc) with severe metric singularities along points and embedded circles, establishing a desingularization process based on work by Li-Mantoulidis in dimension 3. To carry this out, we show that the bordism group of closed 3-manifolds with psc metrics is trivial by explicit methods, using scalar-flat Kähler ALE surfaces recently discovered by Lock-Viaclovsky. This allows us to prove a non-existence result for singular psc metrics on enlargeable 4-manifolds with uniformly Euclidian geometry. As a consequence, we obtain a low-regularity positive mass theorem for asymptotically flat 4-manifolds with non-negative scalar.

Lee Kennard (Syracuse University)
Positive curvature and torus symmetry
In the 1930s, H. Hopf conjectured that an even-dimensional Riemannian manifold with positive sectional curvature has positive Euler characteristic. In joint work with M. Wiemeler and B. Wilking, this is confirmed in the special case where the isometry group has rank at least five. Previous results of this form required the rank to grow to infinity as a function of the manifold dimension. I will outline the proof, with an emphasis on advertising the relatively elementary tools used from geometry, algebraic topology, and representation theory.

Alice Lim (Syracuse University)
Loops to infinity and beyond
The aim of this talk is to discuss relations between Ricci curvature and $n-1$ integer homology. We will see how this relates to the Cheeger-Gromoll Splitting Theorem, and Sormani's Line Theorem.

Daniel Martin (Trinity College)
Rigidity of positive mass theorems in general relativity
Einstein's theory of general relativity states that there is an interaction between the matter in our universe, and the way that it bends. One model of this interaction is given by an asymptotically flat manifold - which models a space with a large isolated object. These metrics decay to the Euclidean metric the further you are from a compact set, and have a geometric invariant known as mass. A famous result called the positive mass theorem of R. Schoen and S.-T. Yau shows that this mass is non-negative when the scalar curvature is non-negative. Furthermore, there is a rigidity result that states when the mass is zero, the space is isometric to Euclidean space.

Another model that has been more recently studied is called an asymptotically hyperbolic manifold - which decays to hyperbolic space. We can similarly define a notion of mass for this class of manifolds, and ask a similar question: Is this mass non-negative when we know something about the scalar curvature, and is the space isometric to hyperbolic space when the mass is zero? In this talk we discuss recent work by L.-H. Huang, Hyun Chul Jang, and the presenter relating to the rigidity portion of this question.

Gideon Maschler (Clark University)
Kähler-Einstein metrics and shear
In past joint work with Amir Aazami, we constructed integrable comlex structures from a

pair of vector fields and a metric on oriented 4-manifolds. The integrability was implied by conditions on the vector fields, in particular one involving their so-called shears. In joint work-in-progress with Robert Ream, we apply this set-up to study local Kähler-Einstein metrics on a class of non-compact 4-manifolds associated with the Lie algebras of the 3-dimensional unimodular groups. We describe the resulting ODE system and survey some solutions.

Arifa Mirza (University of the Punjab)
Bilinearization and soliton solutions of $N = 1$ supersymmetric coupled dispersionless integrable system
An $N = 1$ supersymmetric generalization of coupled dispersionless (SUSY-CD) integrable system has been proposed by writing its superfield Lax representation. It has been shown that under a suitable variable transformation, the SUSY-CD integrable system is equivalent to $N = 1$ supersymmetric sine-Gordon equation. A superfield bilinear form of SUSY-CD integrable system has been proposed by using super Hirota operator. Explicit expressions of superfield soliton solutions of SUSY-CD integrable system have been obtained by using the Hirota method.

Frank Morgan (Williams College)
Double bubbles in space with density
In 2002 and 2008, Hutchings, Morgan, Ritor, Ros, and Reichardt proved the Double Bubble Theorem in Euclidean space: that the standard double bubble consisting of three spherical caps meeting at 120 degrees is the least-perimeter way to enclose and separate two given volumes. In 2018 Milman and Neeman announced a solution in Gauss space (Euclidean space with Gaussian density), and there has been progress for other densities, some by undergraduates, although many questions remain open.

Robert Ream (Clark University)
Weyl-minimal surfaces and the adjunction inequality
Weyl-minimal surfaces are an analog of minimal surfaces for conformal manifolds with a Weyl connection. Many results for minimal surfaces can be adapted to Weyl-minimal sufaces. In particular, for 4-manifolds, there is a non-integrable almost complex structure on the weightless twistor space whose J-holomorphic curves correspond to branched Weyl-minimal surfaces, and when M is almost complex with the canonical Weyl connection branched Weyl-minimal surfaces satisfy the adjunction inequality.

Carlos Vega (Binghamton University)
Splitting spacetime
The singularity theorems of Hawking and Penrose use energy conditions to establish the existence of singularities in large, generic classes of spacetimes. These theorems can be viewed as Lorentzian analogs to Riemannian curvature comparison results like the Bonnet-Myers Theorems. Similarly, the Lorentzian Splitting Theorem is a close spacetime analog of the Cheeger-Gromoll Splitting Theorem for Riemannian manifolds. A related conjecture in the spatially closed setting, known as the Bartnik Splitting Conjecture, remains open. We will discuss some approaches to Lorentzian splitting geometry, including some partial results on this conjecture.

Scott Wilson (Queens College, CUNY)
A generalization of the Kähler symmetries to Hermitian and almost-Kähler manifolds
Among the many striking properties of Kähler manifolds are the so-called Kähler identities,

which are a collection of algebraic relations that hold for the natural differential operators on differential forms. These lead to important connections between geometry and topology, as well as various symmetries and dualities in the Hodge numbers of Kähler manifolds. In this talk I will describe generalizations of these results in two (mirror) directions: the non-integrable (i.e. almost-Kähler) case, and the non-symplectic (i.e. Hermitian) case. In each case we recover all of the expected symmetries, as well as hard Lefschetz duality, on a naturally defined subspace of harmonic differential forms, so that the geometry is bounded above by the topology.

William Wylie (Syracuse University)
Restrictions on topology of extreme black holes
In spacetime dimension four, Hawking showed that the cross sections of an event horizon of asymptotically flat stationary black holes must have spherical topology. In higher dimensions such a simple characterization does not hold as illustrated by "black ring" solutions in spacetime dimension five. A natural question then arises: what are the possible horizon topologies in higher dimensions? In this talk, Ill discuss bounds on the first Betti number and structure results for the fundamental group of horizon cross-sections for extreme stationary vacuum black holes in arbitrary dimension, without additional symmetry hypotheses. The main element of the proof is a generalization of the Cheeger-Gromoll splitting theorem from Riemannian geometry that holds for the universal cover of the near-horizon geometry. This is joint work with Marcus Khuri from Stony Brook and Eric Woolgar from Univ. of Alberta.