Union College Math Conference: Number Theory

September 13–15, 2019

Friday program

4:30–5:30pm: Reception and registration (Olin Rotunda)

5:30-6:30: Plenary talk (Olin 115)
John McCleary: *The Oberwolfach archives and topology*

Saturday program

8:00–9:00am: Coffee & pastries, registration (Olin Rotunda)

9:00–10:30am: Session I (Lippman 017)

- 9:00–9:25 Jay Klangwang: *On the Zeros of Certain Cusp Forms of Weight 3k*
- 9:30–9:55 Brandon Alberts: *Counting Towers of Number Fields*
- 10:00–10:25 Heidi Goodson: *Towards the Sato-Tate Groups of Trinomial Hyperelliptic Curves*

10:30–11:00am: Coffee break (Olin Rotunda)

11:00am–12:00pm: Plenary talk (Olin 115)
Álvaro Lozano-Robledo: *Recent progress in the classification of torsion subgroups of elliptic curves*

12:00–2:00pm: Lunch break

2:00–4:00pm: Session II (Lippman 017)

- 2:00–2:25 Jeff Hatley: *Goldfeld's Conjecture and Iwasawa Theory*
- 2:30–2:55 Doug Haessig: *A relation between symmetric power L-functions of Kloosterman sums and Hecke polynomials*
- 3:00–3:25 Briang Hwang: *Moduli spaces of Galois representations and the structure of their deformation rings*
- 3:30–3:55 Free time (may use Wold 128 for mathematical discussions)

4:00–4:30 pm: Coffee break (Olin Rotunda)

4:30–5:30 pm: Plenary talk (Olin 115)
Robert Ghrist: *Applied Topology: Sheaves and Cosheaves*

6:00-8:30 pm: Conference Dinner (Old Chapel)

Sunday program

4:30–5:30 pm: Plenary talk (Olin 115)
Carolyn Gordon: *Decoding geometry and topology from the Steklov spectrum of orbisurfaces*

Abstracts

Alvaro Lozano-Robledo (University of Connecticut)
Plenary talk: *Recent progress in the classification of torsion subgroups of elliptic curves*
An elliptic curve is a smooth projective curve, of genus 1, with at least one point defined over its field of definition. Elliptic curves are ubiquitous in number theory, algebraic geometry, complex analysis, cryptography, physics, and beyond. One of their most interesting features is that their set of rational points forms a finitely generated abelian group (a fact known as the Mordell–Weil theorem). Little is known about the rank of the group of rational points over a number field but, recently, there has been much progress in the understanding of the points of finite order (a.k.a., torsion points). This talk will be a survey of recent results and methods used in the classification of torsion subgroups of elliptic curves over finite and infinite extensions of the rationals, and over function fields.

Brandon Alberts (University of Connecticut)
Counting Towers of Number Fields

Fix a number field K and a finite transitive subgroup $G \leq S_n$. Malle's conjecture proposes asymptotics for counting the number of G-extensions of number fields F/K with discriminant bounded above by X. A recent and fruitful approach to this problem introduced by Lemke Oliver, Wang, and Wood is to count inductively. Fix a normal subgroup $T \triangleleft G$. Step one: for each G/T-extension L/K, first count the number of towers of fields $F/L/K$ with $\mathrm{Gal}(F/L) \cong T$ and $\mathrm{Gal}(F/K) \cong G$ with discriminant bounded above by X. Step two: sum over all choices for the G/T-extension L/K. In this talk we discuss the close relationship between step one of this method and the first Galois cohomology group. This approach suggests a refinement of Malle's conjecture which gives new insight into the problem. We give the solution to step one when T is an abelian normal subgroup of G, and convert this into nontrivial lower bounds for Malle's conjecture whenever G has an abelian normal subgroup.

Heidi Goodson (Brooklyn College)
Towards the Sato-Tate Groups of Trinomial Hyperelliptic Curves
Let C be a smooth projective curve defined over $\mathbb{Q}$. For primes p of good reduction, we define the trace of Frobenius to be $t_p(C) = p + 1 - \#\overline{C}(\mathbb{F}_p)$, where $\overline{C}$ denotes the reduction

of C modulo p. A theorem of Weil gives the following bound for the trace of Frobenius $|t_p| \leq 2g\sqrt{p}$, where g is the genus of the curve.

Let $x_p = t_p/\sqrt{p}$ denote the normalized trace. Then the Weil bounds tell us that $x_p \in [-2g, 2g]$, and we can look at the distribution of the x_p in this interval as $p \to \infty$. There are many known results for these distributions, called Sato-Tate distributions, for lower genus curves. To determine the distributions, rather than computing the traces of Frobenuis for the curves over $\mathbb{F}_p$ for infinitely many primes p, we instead study the Sato-Tate groups of the Jacobians of the curves.

In this talk, I will discuss Sato-Tate groups of the Jacobian of curves of the form

$$C_1 : y^2 = x^{2g+2} + c, C_2 : y^2 = x^{2g+1} + cx, C_3 : y^2 = x^{2g+1} + c,$$

where g is the genus of the curve and $c \in \mathbb{Q}^*$ is constant. This is joint work with M. Emory and A. Peyrot.

Doug Haessig (University of Rochester)
A relation between symmetric power L-functions of Kloosterman sums and Hecke polynomials.

A conjectural equality between the symmetric power L-functions of the Legendre family of elliptic curves and Hecke polynomials of cusp forms of level Gamma(2) has been known since the 1970s. In this brief talk, we present a conjectural equality between symmetric power L-functions of Kloosterman sums and Hecke polynomials on Gamma1(3) in the case when the prime p = 3. The bridge connecting these two theories will be discussed.

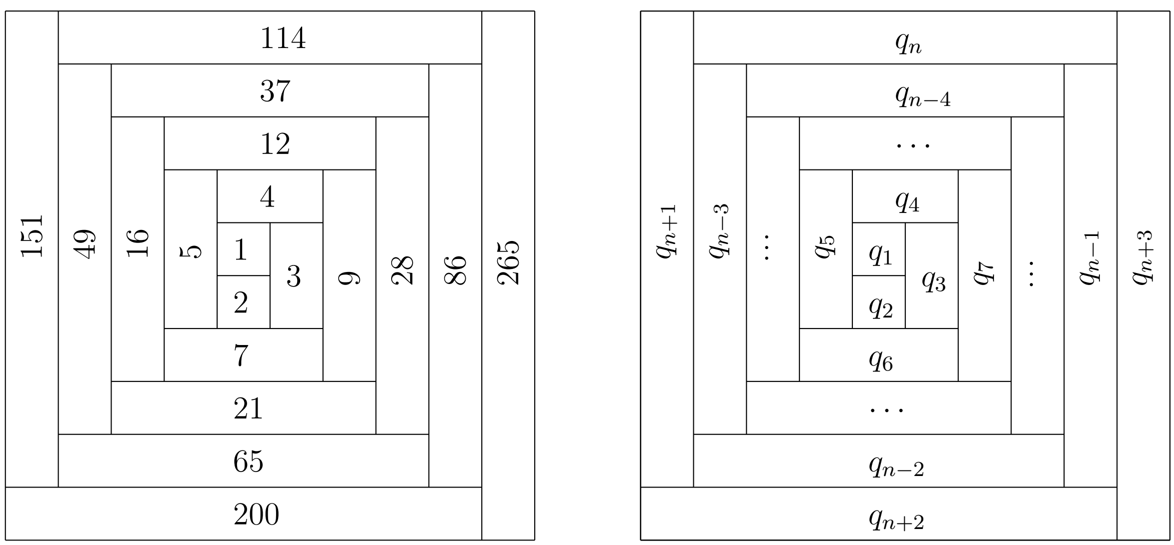
Jeff Hatley (Union College)
Goldfeld's Conjecture and Iwasawa Theory
Fix an elliptic curve $E/\mathbf{Q}$ and consider its family of quadratic twists $\{E^{(d)}\}$. Goldfeld's conjecture is that 50% of these quadratic twists have analytic rank 0 and 50% have rank 1. We will discuss some recent progress on this conjecture due to Kriz and Li which involves a surprising application of Iwasawa theory.

Brian Hwang (Cornell University)
Moduli spaces of Galois representations and the structure of their deformation rings
Galois representations are a common way to study number-theoretic objects like modular forms and algebraic varieties over number fields. A convenient way to parameterize a collection of such representations in a p-adically continuous way is via Galois deformation rings, which are complete noetherian rings whose points corresponds to Galois representations that satisfy certain conditions. We can ask from a naive purely algebraic perspective: what rings arise as Galois deformation rings? We show that even if we only allow for a very restricted class of local conditions, we can cook up a deformation problem whose associated functor is represented by a ring of the desired form (under some mild assumptions). This admits a few different interpretations, and in particular, some curious consequences regarding the geometry of modular curves of square-free level.

Jay Klangwang (Union College)
On the Zeros of Certain Cusp Forms of Weight 3k
F. K. C. Rankin and Swinnerton-Dyer proved that all zeros of the Eisenstein series for the full modular group lie on lower boundary arc of the fundamental domain. In this talk, we

will introduce certain types of cusp forms of weight $3k$ and explore the location of their zeros in the fundamental domain. By extending F.K.C. Rankin and Swinnerton-Dyer argument, we show that most of the zeros in the fundamental domain lie on the boundary of the fundamental domain.

Alexandra Newlon (Colgate University)
Fibonacci Quilt Game
Zeckendorf proved that every positive integer has a unique representation as a sum of non-consecutive Fibonacci numbers. The Zeckendorf game, defined by Baird-Smith, Epstein, Flint, and Miller, is based on this notion of decomposition for the Fibonacci sequence. For an integer n, it begins with n 1's in bin F_1 and bins $F_2, F_3, \ldots$ empty. Two players then take turns combining two consecutive terms or splitting copies of a term and end in a legal decomposition of n. Previous work has found a non-constructive proof of a winning strategy and explored bounds and distribution of game length. We adapt this game to a different sequence, the Fibonacci Quilt sequence, as defined by Catral, Ford, Harris, Miller, and Nelson. The sequence is constructed by adding the next integer which cannot be expressed as the sum of non-adjacent tiles of a log cabin quilt, starting with 1 at the center, shown below:



Legal decomposition for this sequence, referred to as FQ-legal, requires the same non-adjacent quilt behavior. By construction, this decomposition always exists but is not unique. The sequence is eventually dictated by two recurrence relations

$$q_{n+1} = q_n + q_{n-4}, n \geq 6$$
$$q_{n+1} = q_{n-1} + q_{n-2}, n \geq 5,$$

from which we define four rules with additional initial moves. Using a mono-variant, the sum of the square roots of the indices on each move, we prove that under our construction the game always terminates in an FQ-legal decomposition. We prove the minimum length of a game and further investigate the distribution of the length of a random game. We also examine deterministic games and the existence of a potential winning strategy.

This work is joint with Neelima Borade, Catherine Wahlenmayer, Wangqiao Xi, and Steven J. Miller.