# Union College Mathematics Conference: Applied Topology

SEPTEMBER 18–20, 2026

## Friday program

**4:30–5:30pm:** Reception and registration (Olin Rotunda)

**5:30–6:30pm:** Plenary talk I: Carina Curto: *Network Dominoes: Composable Building Blocks for Recurrent Networks* (Olin 115)

## Saturday program

**8:00–9:00am:** Registration, coffee & light breakfast (Bailey 204)

**9:00–10:30am:** Session I (Bailey 201)

- 9:00–9:25 Robin Belton: *Parameter Selection in Mapper Graphs from Three Perspectives*
- 9:30–9:55 Nikola Sadovek: *Vietoris-Rips and Čech Complexes of Spheres in Context*
- 10:00–10:25 Håvard Bakke Bjerkevik: *Reeb Graphs of Sample Thickenings*

**10:30–11:00am:** Coffee break (Olin Rotunda)

**11:00am–12:00pm:** Plenary talk II: Kristine Bauer: *The Categorical Derivative in Functor Calculus* (Olin 115)

**12:00–1:30pm:** Lunch break

**1:30–3:30pm:** Session II (Bailey 201)

- 1:30–1:55 Barbara Giunti: *A Generalization of Vines and Vineyards for Parameterized Persistence*
- 2:00–2:25 Jiajie (Jerry) Luo: *Interval Decomposition of Persistence Modules over a Principal Ideal Domain*
- 2:30–2:55 Henry Ashley: *Counting Barcodes with the Same Betti Curve*
- 3:00–3:25 Ethan Wright: *How Complex Can a Rips Complex Get?*

**3:30–4:00pm:** Coffee break (Olin Rotunda)

**4:00–5:00pm:** Plenary talk III: Julie Blackwood: *Nonlinear Leslie Matrix Models Across Scales: From the Evolution of Bird Song to Cicada Emergence* (Olin 115)

**5:00–6:00 pm:** Reception (Olin Rotunda)

## Sunday program

**8:00–8:30am:** Coffee & light breakfast (Bailey 204)

**9:00–10:30am:** Session III (Bailey 201)

- 9:00–9:25 Tony Xiaochen Xiao: *Constructing Explicit Maps into Eilenberg-MacLane Spaces from Persistence Cohomology*
- 9:30–9:55 Michael Lesnick: *Limit Computation via Minimal Initial Functors*
- 10:00–10:25 Justin Curry: *Stratification Theory for Reinforcement Learning Games*

**10:30–11:00am:** Coffee break (Olin Rotunda)

**11:00am–12:00pm:** Plenary talk: Lan-Hsuan Huang: *Bartnik's Quasi-Local Mass in General Relativity* (Olin 115)

**12:00–1:30pm:** Lunch break

**1:30–3:30pm:** Session IV (Bailey 201)

- 1:30–1:55 Riley Decker: *The Structure of Merge Tree Presentations*
- 2:00–2:25 Tung Lam: *Delaunay Multifiltrations of Functions on Point Clouds*
- 2:30–2:55 Jake Cordes: *Updating Barcode Algorithms*

# Abstracts

**Carina Curto** (Brown University)
Plenary talk: *Network dominoes: composable building blocks for recurrent networks*
Ever since the Hopfield model, recurrent networks have provided a conceptual cornerstone for our understanding of how the brain stores memories and processes information. The nonlinear dynamics, however, are notoriously difficult to understand. Here we take a bottom-up approach, where small network building blocks can be composed to yield complex attractors with predictable dynamics. This approach yields insights into how inhibition and modularity can be used to shape and tame neural activity in real brains. We will also see some implementations of these dynamics in neuromorphic hardware.

**Henry Ashley** (University at Albany, SUNY)
*Counting Barcodes with the Same Betti Curve*
I will discuss an important inverse problem in TDA: How many different barcodes produce the same Betti curve? Equivalently, given a function

$$\beta\colon [n] = \{1 < \cdots < n\} \to \mathbb{Z}_{\geq 0},$$

how many different ways can we write $\beta$ as a sum of indicator functions supported on intervals in $[n]$? Our answer to this question is to connect persistent homology with the study of Kostant's partition function and, surprisingly, enumerative combinatorics for so-called "magic" juggling sequences studied by Ronald Graham and others, proving an equivalence between our inverse problem and corresponding statements in these other two settings. From an applications and statistics point of view, our work provides a quantification of how lossy the TDA pipeline is when moving from persistent homology to persistent Betti numbers.

This is joint work with Håvard Bjerkevik, Justin Curry, Riley Decker, and Robert Green.

**Håvard Bakke Bjerkevik** (University at Albany, SUNY)
*Reeb Graphs of Sample Thickenings*
In topological data analysis, Reeb graphs are usually defined in terms of connected path components of levelsets of functions $f\colon X \to \mathbb{R}$. Instead of defining Reeb graphs in terms of connected components, we show that if $f_1, f_2\colon X \to \mathbb{R}$ are continuous, then

$$d_{\mathrm{I}}(\mathrm{Rb}(X, f_1), \mathrm{Rb}(X, f_2)) \leq \|f_1 - f_2\|_\infty.$$

If one uses path components, this is not always true. This inequality gives a method to transfer reconstruction results similar to the well-known work of Niyogi, Smale, and Weinberger to the setting of Reeb graphs; we show a couple of examples of this. Our work also raises the question of whether using connected components (as opposed to path components) can help circumvent technical annoyances that tend to show up when working with Reeb graphs.

This is joint work with Nello Blaser and Lars M. Salbu.

**Robin Belton** (Vassar College)
*Parameter Selection in Mapper Graphs from Three Perspectives*
The Mapper algorithm is a popular tool in topological data analysis (TDA) for visualization and data exploration with applications ranging from discovering new types of breast cancer to classifying plants. However, it is difficult to use because the user needs to input values for several parameters, and the "best" values are unknown. In this talk, I will discuss how

my collaborators, students, and I have approached parameter selection in this setting from studying three perspectives: the inverse problem, optimization algorithms, and a statistically detecting signal.

**Jake Cordes** (University at Albany, SUNY)
*Updating Barcode Algorithms*
The barcode of a filtration and its associated representative cycles obtained from persistent homology encode rich topological information often useful in data analysis. However, real-world data sets are often dynamic. As data evolves, simplices in the filtration of the complex may be swapped, inserted, or removed. Tracking how barcodes behave on these dynamic data sets reveals crucial information about the changing data. This theory is commonly referred to as the study of Persistence Vineyards. While applying naive algorithms to the removal of simplices is notoriously expensive, recent algorithms like the Simplicial Removal Update Procedure (SiRUP) efficiently handle the removal of simplices. In this talk, we will describe our work in making Persistence Vineyards more computationally efficient. Specifically, we will introduce Co-SiRUP, an adaptation of SiRUP for persistent cohomology, discuss optimized strategies for the order of simplex removal, and demonstrate what work we have done towards handling general $R = DV$ computations.

**Justin Curry** (University at Albany, SUNY)
*Stratification Theory for Reinforcement Learning Games*
In this talk I will use the framework of poset-stratified spaces to study games, where reward can be both discrete and continuous. Following work by Yuliy Baryshnikov, I will show how certain video games naturally give rise to stratified spaces. Surprisingly, when modern neural nets are trained to play these same video games, a similar stratification structure can be observed in their latent representations. Our methods follow recent work by Michael Robinson and others on using Volume Growth Laws to detect non-manifold structure in the token space for LLMs. We expand and strengthen Robinson's analysis by considering non-textual data and prove a realization result for volume growth in a stratified space. If time permits, I will discuss further control theoretic aspects and text data. This is joint work with Brennan Lagasse, Ngoc B. Lam, Gregory Cox, David Rosenbluth, and Alberto Speranzon.

**Riley Decker** (University at Albany, SUNY)
*The Structure of Merge Tree Presentations*
Every free presentation of a persistence module decomposes (up to isomorphism) as the direct sum of a minimal presentation with trivial presentations. For merge trees, presentations are combinatorial rather than algebraic: a presentation of a merge tree $M$ is a pair $(X, f)$, where $X$ is a graph and $f$ is a monotone function on $X$ such that $M \cong \pi_0(S^{\uparrow}(f))$. In this talk, we establish a structure theorem for presentations of merge trees analogous to the one for modules and use it to resolve an open question posed by Cardona and Lam regarding the 1-presentation distance on merge trees. This is joint work with Mike Lesnick.

**Barbara Giunti** (University at Albany, SUNY)
*A Generalization of Vines and Vineyards for Parameterized Persistence*
In this talk, we provide a new construction for studying parameterized persistence, called a canopy. We do this by making a simple but major modification in the persistence bundle

representation information: namely, rather than tracking a point in the persistence diagram, we instead track some choice of pairs of simplices that created said point. This viewpoint is a combinatorial version of tracking the chain complex information rather than just the output of persistence. We show how to construct the canopies from any filtered filtration function, proving, using the algebraic structure of filtered chain complexes, that different choices of pairs result in homeomorphic structures. Finally, we showcase the power of our approach by using canopies to define vines even in the presence of points with multiplicity; to discuss monodromy; and to obtain some immediate results linking non-trivial monodromy in the persistent homology transform with the existence of non-Hausdorff points in the canopy.

**Tung Lam** (University at Albany, SUNY)
*Delaunay Multifiltrations of Functions on Point Clouds*
The Delaunay filtration of a point cloud $X \subset \mathbf{R}^d$ is a central tool of computational topology, justified by its topological equivalence to the offset (union-of-balls) filtration. Given a function on $X$, this extends to a bifiltration of size $O(|X|^{\lceil (d+1)/2 \rceil})$ satisfying the analogous equivalence. Motivated by time-varying data, we extend it further to a pair of functions, obtaining a trifiltration of size $O(|X|^{\lceil (d+1)/2 \rceil + 1})$ with the same property. Both are computed by adapting the Bowyer-Watson algorithm. I will present the constructions, their size and running time bounds, and our implementations, which handle tens of thousands of points in $\mathbf{R}^3$. This is joint work with Ángel Javier Alonso, Michael Kerber, Michael Lesnick, and Abhishek Rathod.

**Michael Lesnick** (University at Albany, SUNY)
*Limit Computation via Minimal Initial Functors*
It is well known that limits can be computed by restricting along an initial functor, and that this often simplifies limit computation. We systematically study the algorithmic implications of this idea for diagrams indexed by a finite poset. We say an initial functor $F\colon C \to D$ with $C$ small is *minimal* if the sets of objects and morphisms of $C$ each have minimum cardinality, among the sources of all initial functors with target $D$. For $Q$ a finite poset or $Q \subseteq \mathbb{N}^d$ an interval (i.e., a convex, connected subposet), we describe all minimal initial functors $F\colon P \to Q$ and in particular, show that $F$ is always a subposet inclusion. We give efficient algorithms to compute a choice of minimal initial functor. In the case that $Q \subseteq \mathbb{N}^d$ is an interval, we give asymptotically optimal bounds on $|P|$, the number of relations in $P$ (including identities), in terms of the number $n$ of minima of $Q$: We show that $|P| = \Theta(n)$ for $d \leq 3$, and $|P| = \Theta(n^2)$ for $d > 3$. We apply these results to give new bounds on the cost of computing $\lim G$ for a functor $G\colon Q \to \mathbf{Vec}$ valued in vector spaces. For $Q$ connected, we also give new bounds on the cost of computing the *generalized rank* of $G$ (i.e., the rank of the induced map $\lim G \to \operatorname{colim} G$), which is of interest in topological data analysis.

**Jiajie Luo** (University of Chicago)
*Interval Decomposition of Persistence Modules over a Principal Ideal Domain*
The study of persistent homology is largely built upon the structure theorem for persistence modules over a field, which guarantees a decomposition into simple interval modules. However, such a decomposition may fail to exist under more general coefficients. In this talk we show that, given a pointwise free and finitely-generated persistence module over a principal ideal domain, an interval decomposition exists if and only if the cokernel of every structure map is a free module. We first show the existence of an interval decomposition in the setting of

finitely-indexed persistence modules, where we iteratively construct an interval decomposition one module at a time. We also formulate a polynomial-time algorithm that computes an interval decomposition or certifies that no decomposition exists. We then extend our work to the setting of infinitely-indexed persistence modules. This is joint work with Gregory Henselman-Petrusek.

**Nikola Sadovek** (Carnegie Mellon University)
*Vietoris-Rips and Čech Complexes of Spheres in Context*
Vietoris–Rips and Čech complexes whose vertex set is the entire sphere exhibit rich and interesting structure, yet their homotopy types at large parameter scales remain poorly understood. We will explain how the study of these homotopy types, already in the case of the circle, leads naturally to problems in discrete potential minimization, whose extremizers detect the critical scales at which homotopical changes occur. We will then outline how such an understanding of these complexes for higher-dimensional spheres yields quantitative results in geometry and topology, such as Gromov–Hausdorff distance estimates and quantitative nonembeddability results for manifolds.

**Ethan Wright** (University at Albany, SUNY)
*How Complex Can a Rips Complex Get?*
Vietoris-Rips complexes are a widely used construction in topological data analysis, and specifically persistence theory, with several applications. In computing the persistent homology in dimension $d$ of a Vietoris-Rips complex, one must reduce the boundary matrix in the same dimension. The worst-case complexity of the algorithm used to reduce a boundary matrix is $O(N^3)$, where $N$ is the number of simplices in dimension $d+1$. However, the runtime behavior in applications has been observed to be approximately linear. It remains open whether the worst case can be achieved with a Vietoris-Rips filtration. This talk will introduce the construction of Vietoris- Rips complexes and the barcode reduction algorithm. We will then describe recent worst-case constructions which are not Vietoris-Rips, discuss the obstructions to generalizing such constructions to the Vietoris-Rips case, and present some preliminary results on the worst-case bounds for Vietoris-Rips filtrations.

**Tony Xiaochen Xiao** (Northeastern University)
*Constructing Explicit Maps into Eilenberg-MacLane Spaces from Persistence Cohomology*
Persistent cohomology has become a powerful tool for recovering the topology of an underlying space from sampled data. But once a persistent cohomology class has been detected, what topological information does it actually represent? Can it be realized as an explicit geometric object? In this talk, I will introduce Eilenberg–MacLane coordinates—explicit maps from data into Eilenberg–MacLane spaces $K(G,n)$ representing persistent cohomology classes. Using sheaf theory, I construct such maps in arbitrary dimensions $n$ and for arbitrary discrete Abelian coefficient groups $G$, extending circular, projective, and lens coordinates within a unified framework. The construction is fully explicit, yielding recursive formulas via inverses of Čech coboundary operators, and produces new coordinate types, including the first example of $K(\mathbf{Z}/2,2)$-coordinates.