1. Compute \( \iint_R x^2 \, dA \) where \( R \) is the region shown:

Note that the points of intersection of the two graphs are (0,0) and (2,16). Then,

\[
\iint_R x^2 \, dA = \int_{x=0}^{x=2} \int_{y=x^4}^{y=4x^2} x^2 \, dy \, dx = \\
\int_{x=0}^{x=2} x^2 \bigg|_{y=x^4}^{y=4x^2} \, dx = \\
\int_{x=0}^{x=2} (4x^4 - x^6) \, dx = \\
\left( \frac{4x^5}{5} - \frac{x^7}{7} \right) \bigg|_{x=0}^{x=2} = \\
\frac{256}{35} - \frac{128}{7} = \\
\frac{128}{5} \cdot \frac{1}{7} = \\
\frac{128}{35}
\]

2. Evaluate the double integral \( \iint_R \frac{1}{1+y} \, dA \), where \( R \) is the triangular region with vertices (0,0), (2,1), and (0,1). (Hint: Think carefully about the order of integration.)

Note that the line determined by (0,0) and (2,1) has equation \( y = \frac{1}{2} x \), or \( x = 2y \).

Then,

\[
\iint_R \frac{1}{1+y} \, dA = \int_{y=0}^{y=1} \int_{x=2y}^{x=1} \frac{1}{1+y} \, dx \, dy = \\
\int_{y=0}^{y=1} \int_{x=2y}^{x=1} \frac{1}{1+y} \, dx \, dy = \\
\int_{y=0}^{y=1} \frac{1}{1+y} \bigg|_{x=2y}^{x=1} \, dy = \\
\ln(1+y) \bigg|_{y=0}^{y=1} = \\
\ln 2 - \ln 1 = \\
\ln 2
\]
3. Consider the equation in polar coordinates $r = 3 + 3 \cos \theta$ for $0 \leq \theta \leq 2\pi$.

   a. Complete the chart below and then use this information to sketch the graph.

   $\theta$ : $0 \rightarrow \frac{\pi}{2} \rightarrow \pi \rightarrow \frac{3\pi}{2} \rightarrow 2\pi$

   $\cos \theta$ : $1 \rightarrow 0 \rightarrow -1 \rightarrow 0 \rightarrow 1$

   $3\cos \theta$ : $3 \rightarrow 0 \rightarrow -3 \rightarrow 0 \rightarrow 3$

   $r = 3 + 3 \cos \theta$ : $6 \rightarrow 3 \rightarrow 0 \rightarrow 3 \rightarrow 6$

   

   b. Compute the area of the region enclosed by your figure in part a.

   \[
   A = \int_{\theta = 0}^{\theta = 2\pi} \int_{r = 0}^{r = 3 + 3\cos \theta} r \, dr \, d\theta = \\
   \int_{\theta = 0}^{\theta = 2\pi} \int_{r = 0}^{r = 3 + 3\cos \theta} \theta \, d\theta = \\
   \frac{1}{2} \left( \frac{27}{2} + 18 \cos \theta + \left( \frac{9}{2} \right) \cos 2\theta \right) \theta \bigg|_{\theta = 0}^{\theta = 2\pi} = \\
   \frac{1}{2} \left( \frac{27}{2} \right) (2\pi) = \\
   \frac{27\pi}{2}
   \]
4. Set up, but do not evaluate, an integral to determine the volume of the solid bounded above by the surface \( f(x,y) = \sqrt{x^4 + y^6} \), below by the plane \( z=0 \), and laterally by \( y = -3x \), \( y = x \), and \( y = 8 \). (Your answer should include the integrand and limits of integration.)

The region over which we are integrating is shown to the right. Note that the point of intersection of \( y=-3x \) and \( y=8 \) is \((-8/3,8)\) and the point of intersection of \( y=x \) and \( y=8 \) is \((8,8)\).

Then,

\[
V = \int_{y=0}^{y=8} \int_{x=-\frac{y}{3}}^{x=y} \sqrt{x^4 + y^6} \, dx \, dy
\]

5. Consider the integral \( \int_{y=0}^{y=6} \int_{x=y/3}^{x=2} e^{-x^2} \, dx \, dy \).

a. Sketch the region in the xy plane over which the integral is taken.

b. Compute the integral.

\[
\begin{align*}
\int_{y=0}^{y=6} \int_{x=y/3}^{x=2} e^{-x^2} \, dx \, dy &= \\
\int_{x=y/3}^{x=2} \int_{y=0}^{y=3x} e^{-x^2} \, dy \, dx &= \\
\int_{x=0}^{x=2} \left[ ye^{-x^2} \right]_{y=0}^{y=3x} \, dx &= \\
\int_{x=0}^{x=2} \left[ 3x e^{-x^2} \right] \, dx &= (\text{substitution}) \\
\left( -\frac{3}{2} \right) e^{-x^2} \bigg|_{x=0}^{x=2} &= \\
\left( -\frac{3}{2} \right) (e^{-4} - 1) &= \\
\left( \frac{3}{2} \right) (1 - \frac{1}{e^4}) &
\end{align*}
\]
6. Consider the integral \( \int_{y=0}^{y=2} \int_{x=y}^{x=\sqrt{8-y^2}} \sqrt{x^2 + y^2} \, dx \, dy \).

a. Sketch the region in the xy plane over which the integral is taken.

b. Compute the integral.

\[
\int_{y=0}^{y=2} \int_{x=y}^{x=\sqrt{8-y^2}} \sqrt{x^2 + y^2} \, dx \, dy = \int_{\theta=0}^{\theta=\pi/2} \int_{r=0}^{r=\sqrt{8}} r^2 \, dr \, d\theta = \int_{\theta=0}^{\theta=\pi/4} \int_{r=0}^{r=\sqrt{8}} \frac{r^3}{3} \, dr \, d\theta = \left( \frac{8\sqrt{2}}{3} \right) \left( \frac{\pi}{4} \right) = \frac{2\sqrt{2}\pi}{3}.
\]