Magnetic Fields and Magnetic Forces

Overview of magnetism:
Magnetism was known as early as the 13th century BC, where compass needles were already in use in China – its invention probably due to the Arabs or Indians. Magnetite (Fe₃O₄), a naturally occurring mineral, is magnetic and was the first material used as a magnet for human purposes.

Magnetite is actually found in some living materials – some bacteria contain linear chains of magnetite which acts to steer these bacteria towards the earth’s magnetic north. Since magnetic north actually has a downward component in the northern hemisphere, these bacteria use their magnets to find the bottom of the lakes and ponds in which they are found – they are bottom feeders. They were first discovered in the 1970’s by a graduate student at Woods Hole Oceanographic Institute during a summer course. This same student then wrote a proposal to go to the southern hemisphere and look for magnetic bacteria there. In the southern hemisphere these bacteria use the magnetic force to come to the surface – they are surface feeders. Needless to say he was successful in this and these bacteria have been studied in detail.

These magnetic minerals act as compass needles and each has two poles, which we term north and south. Just as with electric charge, like poles repel one another and unlike poles attract. It is these forces that hold refrigerator magnets up. A magnet with north and south poles will feel a force in the earth’s magnetic field. If a bar magnet – compass needle – is suspended from its mid-point and allowed to swivel, it will orient with its north pole pointing towards the earth’s geographic north pole. This geographic north pole is actually a magnetic south pole. Similarly the bar magnet’s south pole is attracted to the earth’s geographic south pole which is a magnetic north pole.

It was found by William Gilbert in the late 1500’s that magnetic poles exert forces that follow an inverse square law just like the force between electric charges. The major difference is that there is no such thing as an isolated magnetic pole. Magnetic poles are always found in pairs of equal and opposite strength – known as magnetic dipoles. So, there is no direct analogy to electric charge – magnetic charge – or an isolated monopole – does not exist.

We will see that magnetic fields are really due to electric currents – whether microscopic or macroscopic and that even the natural magnetism of magnetite is due to microscopic electric current flow. For now, we ignore how a magnetic field is produced, but look at the effects of a magnetic field on electric charges.

We will describe the magnetic field with the symbol \( \vec{B} \) and mappings of the magnetic field will use similar pictures to the mapping for the electric field.

Suppose that there is a uniform magnetic field in a region of space (ignoring for now, how it is made and the electric currents needed to produce it). A charge \( q \) located in this region of space may feel a new (to us) force, called the magnetic force \( \vec{F}_B \), but the force law is more complex than that for the electric force on the charge \( q \) (given by \( \vec{F}_E = q\vec{E} \)).

- First, it is found that if the charge \( q \) is stationary, there is no magnetic force on it (if the velocity \( v = 0 \), then \( \vec{F}_B = 0 \))
- If the charge \( q \) moves in the direction of the magnetic field, then there is still no magnetic force on the charge (if \( \vec{v} \parallel \vec{B} \) then \( \vec{F}_B = 0 \))
• If the charge \( q \) has a velocity component perpendicular to the magnetic field, then there is a magnetic force on \( q \) and it is given, in magnitude, by \( F_B = qv \cdot B \), where \( v \cdot B \) is the component of the velocity perpendicular to \( B \).

• The direction of the force is more complex – it is perpendicular to the plane containing the velocity and magnetic field vectors and given by a right-hand rule to choose which of the two possible directions perpendicular to the plane containing those two vectors the force has. (See text for this discussion)

• In general, finally, given a charge \( q \) moving with a velocity \( \vec{v} \), the magnetic force is given by:
  \[ \vec{F}_B = q\vec{v} \times \vec{B}, \]
  where the \( \times \) represents the vector cross product between the two vectors. Recall that in magnitude this means that \( F_B = qvB\sin\theta \), where the angle \( \theta \) is that between the velocity and magnetic field.

The SI units for \( B \) are given from this equation in tesla, where 1 T = 1 N/(C m/s) or 1 T = 1 N/(A m). Another commonly used unit for \( B \) is the gauss, where 1 G = 10^{-4} T, that is the gauss is a much smaller unit of \( B \). The largest man-made magnets are only around 30 T, so the T is a very large unit of \( B \).

Example: Force on and trajectory of an electron in a uniform magnetic field. With velocity perpendicular to \( B \) field: 
\[ F = evB = ma = mv^2/r. \]
If electron has gained its velocity by accelerating from rest through a potential \( V \) then we have that 
\[ eV = \frac{1}{2} mv^2. \]
Combining these two equations, we can solve for \( v \) in the second and substitute into the first: 
\[ v^2 = 2eV/m, \]
so 
\[ eB = mv/r = m[2eV/m]^{1/2}/r. \]
Squaring the equation and solving for \( e/m \), we find 
\[ e^2B^2 = m^2[2eV/m]/r^2, \]
and solving for \( e/m \) we have 
\[ e/m = 2V/(B^2r^2). \]
This equation will be used in lab tomorrow, together with an expression for the uniform \( B \) field to measure the charge-to-mass ratio on the electron.