1. Let $P = \{1, 2, 3, 4, 5, 8, 30, 90, 120\}$ ordered by divides.
   (a) Draw a Hasse diagram for $P$.
   (b) Find $\text{lub}\{2, 3\}$, $\text{lub}\{2, 90\}$, $\text{lub}\{4, 5\}$, $\text{lub}\{90, 120\}$, and $\text{lub}\{3, 4, 5\}$.
   (c) Find $\text{glb}\{2, 3\}$, $\text{glb}\{2, 8\}$, $\text{glb}\{3, 4\}$, $\text{glb}\{4, 30\}$, and $\text{glb}\{90, 120\}$.

2. Consider $\mathcal{P}(\{1, 2, 3, 4, 5, 6\})$ ordered by $\subseteq$, and the subsets $S_1 = \{\{1\}, \{2, 3\}\}$, $S_2 = \{\{1\}, \{1, 3\}\}$, $S_3 = \{\{1, 2\}, \{3, 4\}\}$, and $S_4 = \{\{1, 2\}, \{2, 3\}, \{2, 6\}\}$.
   (a) Find the least upper bound of $S_i$, for $i = 1, 2, 3, 4$.
   (b) Find greatest lower bound $S_i$, for $i = 1, 2, 3, 4$.

3. Prove that if the greatest lower bound of $S$ exists in a poset $P$, then it is unique (i.e., show that if $x_1$ and $x_2$ are greatest lower bounds of $S$, then $x_1 = x_2$).

4. Give an example of a poset $P$ which has an antichain $\{x, y, z\}$ such that $\text{lub}\{x, y, z\}$ exists, but $\text{lub}\{x, y\}$, $\text{lub}\{x, z\}$, and $\text{lub}\{y, z\}$ do not exist.

5. Among the many isomorphism classes of posets with 5 elements, there are 5 which are lattices. Draw a Hasse diagram for each of these lattices with 5 elements.