Propositions, Theorems, and Corollaries on Size of Sets

Proposition 1: \( \cong \) is an equivalence relation.

Proposition 2: If \( X \cong A \) and \( Y \cong B \), then \( X \times Y \cong A \times B \).

Proposition 3: \( X \not\cong P(X) \)

Proposition 4: If \( X \cong Y \) and \( Y \) is countable, then \( X \) is countable.

Proposition 5: If \( f: X \to Y \) is 1-1 and \( A \subseteq X \), then \( A \cong f(A) \).

Theorem 1: \( \mathbb{N} \times \mathbb{N} \) is countable.

Corollary: \( \mathbb{Z} \times \mathbb{N} \) is countable.

Theorem 2: Every infinite subset of \( \mathbb{N} \) is countable.

Corollary: Every infinite subset of a countable set is countable.

Theorem 3: \( \mathbb{Q} \) is countable.

Theorem 4: \( \mathbb{R} \) is uncountable.