Propositions, Theorems, and Corollaries on Size of Sets

Proposition 1: $\cong$ is an equivalence relation.

Proposition 2: If $X \cong A$ and $Y \cong B$, then $X \times Y \cong A \times B$.

Proposition 3: $X \ncong \mathcal{P}(X)$

Proposition 4: If $X \cong Y$ and Y is countable, then X is countable.

Proposition 5: If $f\colon X \to Y$ is $1-1$ and $A \subseteq X$, then $A \cong f(A)$.

Theorem 1: $\mathbb{N} \times \mathbb{N}$ is countable.

Corollary: $\mathbb{Z} \times \mathbb{N}$ is countable.

Theorem 2: Every infinite subset of $\mathbb{N}$ is countable.

Corollary: Every infinite subset of a countable set is countable.

Theorem 3: $\mathbb{Q}$ is countable.

Theorem 4: $\mathbb{R}$ is uncountable.