

1. Let  $P = \{1, 2, 3, 5, 30\}$  ordered by divides.
  - (a) Draw the Hasse diagram for  $P$ .
  - (b) Find the chromatic number  $\chi(G)$ , where  $G$  is the Hasse diagram for  $P$ .
  - (c) Use a counting argument to find the chromatic polynomial  $P(G, t)$ , where  $G$  is the Hasse diagram for  $P$ .
  - (d) Use the Edge Theorem to find  $P(G, t)$ , where  $G$  is the Hasse diagram for  $P$ .
2. Let  $P = \{3, 6, 15, 12, 30, 60\}$  ordered by divides.
  - (a) Draw the Hasse diagram for  $P$ .
  - (b) Find  $\chi(G)$ , where  $G$  is the Hasse diagram for  $P$ .
  - (c) Use the Union Theorem to find  $P(G, t)$ , where  $G$  is the Hasse diagram for  $P$ .
3. Show that  $K_n$  has an Eulerian Circuit  $\iff n$  is odd.
4. Draw a Hasse diagram for each isomorphism class of posets with 4 elements.  
Hint: There are 16 isomorphism classes.
5. Let  $P = \{2, 3, 4, \dots, 100\}$  ordered by divides. Find  $\text{Max}(P)$  and  $\text{Min}(P)$ .
6. Let  $\mathbb{Z}$  denote the set of integers, and define

$$a \sqsubseteq b \iff b - a \text{ is a nonnegative even integer}$$

- (a) Show that  $\sqsubseteq$  is a partial order of  $\mathbb{Z}$ .
  - (b) Draw a Hasse diagram for the poset  $(\mathbb{Z}, \sqsubseteq)$ .
7. Given posets  $P$  and  $Q$ , let  $P \times Q$  be the poset defined in Graded Homework 4.  
Prove:  $(x', y')$  covers  $(x, y) \iff (x' = x \text{ and } y' \text{ covers } y) \text{ or } (x' \text{ covers } x \text{ and } y' = y)$ .