Math 219 Practice Problems for Test 1

Part I. Use a counting argument to justify each of the following.

1. How many lists of length 12 formed from the numbers 0–9 contain exactly one 3 or one 7 but not both?
2. How many non-repeating lists of length 7 formed from the numbers 0–9 contain at least one of 3 and 7?
3. How many n-digit positive numbers contain at least one 5? Note that leading zeroes are not allowed.
4. How many arrangements of the letters A–H do not contain the word BAD?
5. How many non-repeating lists of length 5 using the letters A–H do not have two vowels in a row?
6. How many lists of length 10 using the letters A–F contain exactly four A's?

Part II. Prove each of the following.

1. Suppose $\mathcal{G}$ is a graph of order n and every vertex of $\mathcal{G}$ has degree at least $\frac{n-1}{2}$. Prove that $\mathcal{G}$ is connected.
2. Suppose every vertex of $\mathcal{G}$ has degree at least n, where $n \geq 1$. Prove that $\mathcal{G}$ has a path of length n with no repeated vertices.

Answers to Part I.

1. $24 \cdot 8^{11}$
2. $\frac{10!}{3!} - 8!$
3. $9 \cdot 10^{n-1} - 8 \cdot 9^{n-1}$
4. $8! - 6 \cdot 5!$
5. $\frac{8!}{3!} - 8 \cdot \frac{6!}{3!}$
6. $\binom{10}{4} \cdot 5^6$