1. Let \( P = \{1, 2, 3, 4, 5, 8, 30, 90, 120\} \) ordered by divides.

   (a) Draw a Hasse diagram for \( P \).

   (b) Find \( \text{lub}\{2, 3\} \), \( \text{lub}\{2, 90\} \), \( \text{lub}\{4, 5\} \), and \( \text{lub}\{3, 4, 5\} \).

   (c) Find \( \text{glb}\{2, 3\} \), \( \text{glb}\{2, 8\} \), \( \text{glb}\{3, 4\} \), \( \text{glb}\{4, 30\} \), and \( \text{glb}\{3, 4, 5\} \).

2. Consider \( \mathcal{P}(\{1, 2, 3, 4, 5, 6\}) \) ordered by \( \subseteq \), and the subsets \( S_1 = \{\{1\}, \{2, 3\}\} \), \( S_2 = \{\{1\}, \{1, 3\}\} \), \( S_3 = \{\{1, 2\}, \{3, 4\}\} \), and \( S_4 = \{\{1, 2\}, \{2, 3\}, \{2, 6\}\} \).

   (a) Find the number of elements in the set of upper bounds of \( S_i \), for \( i = 1, 2, 3, 4 \).

   (b) Find the least upper bound of \( S_i \), for \( i = 1, 2, 3, 4 \).

   (c) Find greatest lower bound \( S_i \), for \( i = 1, 2, 3, 4 \).

3. Give an example of a poset \( P \) which has an antichain \( \{x, y, z\} \) such that \( \text{lub}\{x, y, z\} \) exists, but \( \text{lub}\{x, y\} \), \( \text{lub}\{x, z\} \), and \( \text{lub}\{y, z\} \) do not exist.

4. Prove that if \( P \) is a lattice, then \( \text{lub}\{x_1, \ldots, x_n\} \) exists, for all \( x_1, \ldots, x_n \in P \).

5. Prove that if the greatest lower bound of \( S \) exists in a poset \( P \), then it is unique (i.e., show that if \( x_1 \) and \( x_2 \) are greatest lower bounds of \( S \), then \( x_1 = x_2 \)).