Use a counting argument to establish each of the following. Make sure you justify each step of your argument.

1. How many lists of length 2 formed from 0, 1, . . . , 9 have exactly one 5?
2. How many lists of length 3 formed from 0, 1, . . . , 9 have exactly one 5?
3. How many 3-digit numbers contain exactly one 5?
4. How many 3-digit numbers contain at least one 5?
5. How many 4-digit numbers contain exactly one 0?
6. How many 4-digit numbers contain at least one 0?
7. How many 10-letter sequences can be formed from A, B, C, D, E?
8. How many 10-letter sequences containing at least one vowel can be formed from A, B, C, D, E?
9. The special of the day at a local shop consists of a large dish of vanilla ice cream topped with your choice of one fruit topping (bananas, blueberries, raspberries, or strawberries), one type of nuts (almonds, peanuts, or walnuts), and one sauce (fudge or caramel). Whipped cream is optional. How many different selections are possible?
10. Repeat 9 assuming that fruit, nuts, and sauce are each optional.

Answers:
1. 18 2. 243 3. 225 4. 252 5. 2187 6. 2439 7. $5^{10}$ 8. $5^{10} - 3^{10}$ 9. 48 10. 120