Math 99: Homework Sheet 7

In 1 – 4, do (a) – (d) for the given relation $R$ on the set $X = \{1, 2, 3, 4, 5\}$.

Determine if $R$ is:
(a) reflexive
(b) symmetric
(c) transitive
(d) an equivalence relation

If $R$ is an equivalence relation, find: (i) the equivalence classes, and (ii) $X/R$.
If not, find the smallest equivalence relation $R'$ such that $R \subseteq R'$.

1. $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 3), (3, 1)\}$
2. $R = \{(2, 2), (3, 3), (1, 2), (1, 3), (1, 5)\}$
3. $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 3), (1, 4), (2, 5), (3, 1), (3, 4), (4, 1), (4, 3), (5, 2)\}$
4. $R = \{(m, n) \in X \times X \mid m \text{ divides } n\}$

In 5 – 7, find an example of a relation $R$ on $\{1, 2, 3\}$ satisfying the given properties.

5. $R$ is reflexive and symmetric, but not transitive.
6. $R$ is reflexive and transitive, but not symmetric.
7. $R$ is symmetric and transitive, but not reflexive.

8. Determine if $P$ is a partition of $\{1, 2, 3, 4\}$.
   (a) $P = \{\{1, 2\}, \{3, 4\}\}$  (b) $P = \{\{1, 2\}, \{4\}\}$  (c) $P = \{\{1, 2\}, \{2, 3, 4\}\}$

9. Find $R_P$, for the given partition $P$ of $\{v, w, x, y, z\}$.
   (a) $P = \{\{v, w\}, \{x\}, \{y, z\}\}$  (b) $P = \{\{v\}, \{w\}, \{x, y, z\}\}$

10. How many equivalence relations on $\{a, b, c, d\}$ have exactly two equivalence classes? Justify that you have given all of them. Hint: Use partitions.
Answers

1. (a) yes; (b) yes; (c) yes; (d) yes; (i) \([1] = [3] = \{1, 3\}, [2] = \{2\}, [4] = \{4\}, [5] = \{5\}\);
   (ii) \(X/R = \{\{1, 3\}, \{2\}, \{4\}, \{5\}\}\).

2. (a) no; (b) no; (c) yes; (d) no; \(R' = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 2), (1, 3),
   (1, 5), (2, 1), (2, 3), (2, 5), (3, 1), (3, 2), (3, 5), (5, 1), (5, 2), (5, 3)\}\).

3. (a) yes; (b) yes; (c) yes; (d) yes;
   (i) \([1] = [3] = [4] = \{1, 3, 4\}, [2] = [5] = \{2, 5\}\); (ii) \(X/R = \{\{1, 3, 4\}, \{2, 5\}\}\).

4. (a) yes; (b) no; (c) yes; (d) no; \(R' = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}\).

5. \(R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}\) is one of several examples.

6. \(R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}\) is one of several examples.

7. \(R = \{(1, 1)\}\) and \(R = \{(1, 1), (1, 2), (2, 1), (2, 2)\}\) are two of several examples.

8. (a) yes; (b) no; (c) no.

9. (a) \(R_P = \{(v, v), (w, w), (v, w), (w, v), (x, x), (y, y), (z, z)\}\)
   (b) \(R_P = \{(v, v), (w, w), (x, x), (y, y), (z, z), (x, y), (x, z), (y, x), (y, z), (z, x), (z, y)\}\)

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