Directions: You may use a non-programmable, non-graphing scientific calculator. Show all your work.

Summation formulas:
\[ \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \]
\[ \sum_{k=1}^{n} k^2 = \frac{1}{6} n(n+1)(2n+1) \]
\[ \sum_{k=1}^{n} k^3 = \left[ \frac{n(n+1)}{2} \right]^2 \]

1. a) The left-hand graph shows the graph of \( f(x) \). On the right-hand graph, sketch the graph of \( f^{-1}(x) \).

   ![Graph](image)

   b) How is the number \( e \) chosen (defined)?

   The number \( e \) is chosen so that the slope of ____________________________

2. Differentiate each of the following.
   a) \( y = e^{\sqrt{x}} \tan(x) + e^{\tan(x)} \)

   b) \( y = x^{17} \sin(x) \)
3. Differentiate each of the following. You need not simplify your answers.
   a) \( y = \log_2 (1 - x^3) \)
   b) \( y = \ln \left( \frac{(x^4 - x)^5 \sqrt{\sin(x)}}{x^{3/4}} \right) \)

4. The count in a bacteria culture was 300 after 10 minutes and 800 after 25 minutes. What was the initial size of the culture? (Not covered in Winter 2009.)

5. Do EITHER a) or b). Either one should take 4-6 lines of calculation with a few words of explanation.
   a) Use the limit definition of derivative to prove that \( \frac{d}{dx}(e^x) = e^x \).
   b) Prove that \( \sum_{k=1}^{n} (a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k \)

6. a) Is the following statement true? Support your answer with a calculation.
   \[ \int \frac{1}{x^2 - 16} \, dx = \frac{1}{8} \ln \left( \frac{x - 4}{x + 4} \right) + C \]
   b) Geometrically, what does \( \int_{a}^{b} f(x) \, dx \) represent? (i.e. what is the informal, geometric definition of the definite integral? 2 sentences and a picture, which should include the graph of a function that goes both above and below the \( x \)-axis.)

7. Evaluate each of the following.
   a) \( \int \sin(x) \cos(x) \, dx \)
   b) \( \int -5x^2 e^{-2x^3} \, dx \)

8. a) Use sigma notation to write the sum: \( \frac{1}{(6)^2} + \frac{1}{(8)^2} + \frac{1}{(10)^2} + ... + \frac{1}{(26)^2} \)
   b) Find the numerical value of the following sum: \( \sum_{k=1}^{50} k(k - 1)(k + 1) \)

9. a) Evaluate the definite integral \( \int_{-2}^{6} -x \, dx \) by using a geometric formula. Include a sketch.
   b) Approximate the definite integral \( \int_{1}^{3} x^2 + 1 \, dx \) using \( n = 4 \) subintervals and right-hand endpoints to determine the height of the rectangles. Sketch the region of the area and the approximating rectangles. From your sketch, decide whether your approximation is an underestimate or an overestimate of \( \int_{1}^{3} x^2 + 1 \, dx \).

10. Use the definition of the definite integral as a limit of Riemann sums to evaluate \( \int_{1}^{2} 3x + 1 \, dx \).