# Union College Mathematics Conference
# Abstracts for the Algebraic Topology Session
# October 19-20, 2013

**Michael Andrews (MIT)**
**The $v_1$-periodic homotopy of the sphere spectrum and the classical Adams spectral sequence**

Abstract: The $v_1$-periodic homotopy groups of the sphere spectrum are well known and are closely related to the image of the (stable) $J$-homomorphism. Working at an odd prime I will outline what I hope is a new method for computing these groups. As a result of the computation we will discover infinitely many differentials in the classical mod $p$ Adams spectral sequence.

**Gunnar Carlsson (Stanford University)**
**The Shape of Data**

Abstract: In recent years, a recognition has been developing that notions of shape can serve as a valuable paradigm for organizing and understanding large and complex data sets. Topology, which is the mathematical discipline which concerns itself with the study of shape, can therefore be adapted to the study of such data sets. We will talk about a number of such methods, with examples.

**Michael Ching (Amherst College)**
**Connections between the homotopy and manifold calculi of functors**

Abstract: There are two interesting theories for studying the calculus of functors on various categories of topological spaces. The Goodwillie-Weiss manifold calculus concerns contravariant functors on the category of open subsets of a manifold $M$. On the other hand, Goodwillie's homotopy calculus can be used to study covariant functors on the category of all pointed spaces. A key observation is that these two types of functors can be related via the operation of one-point compactification.

I will explore some connections between these theories that arise from recent joint work with Greg Arone. The basic slogan is that the homotopy calculus of functors from pointed spaces to spectra is the limit of the (stable) manifold calculus for open subsets of Euclidean space $\mathbf{R}^m$, as $m$ tends to infinity. The goal of the talk will be to make this statement precise.

**Michael Donovan (MIT)**
**The Adams spectral sequence for simplicial algebras**

Abstract: We discuss an Adams spectral sequence for simplicial commutative augmented $F_2$-algebras, whose $E_2$ page depends functorially on the Andre-Quillen cohomology. Using Koszul methods we show that it admits a vanishing line from $E_2$, whenever the simplicial algebra in question is connected.

**Tom Fiore (University of Michigan, Dearborn)**
**Waldhausen Additivity and Approximation in Quasicategorical $K$-Theory**

Abstract: In this talk, I will present two classical theorems of algebraic topology in the new context of quasicategories: Waldhausen Additivity and Approximation.

Waldhausen Additivity, in its most general form, says that Waldhausen $K$-theory sends a split-exact sequence $\mathcal{A} \to \mathcal{E} \to \mathcal{B}$ to a stable equivalence $K(\mathcal{E}) \to K(\mathcal{A}) \vee K(\mathcal{B})$ of spectra. I will sketch a proof for the case that $\mathcal{A}$, $\mathcal{E}$, and $\mathcal{B}$ are Waldhausen quasicategories satisfying mild hypotheses. This is joint work with Wolfgang Lück. The method here is to prove the classical theorem in a mostly simplicial way, combining elements of previous proofs, and then carry this proof over to quasicategories.

Waldhausen Approximation, on the other hand, provides one answer to the question: when does an equivalence of homotopy categories induce an equivalence of $K$-theory spectra? In the quasicategorical context, it suffices for an exact functor to reflect cofibrations and induce an equivalence of homotopy categories.

**John Harper (Ohio State University, Newark)**
**$K$-coalgebras, $TQ$-completion, and a structured ring spectra analog of Quillen–Sullivan theory**

Abstract: An important theme in current work in homotopy theory is the investigation and exploitation of enriched algebraic structures on spectra that naturally arise, for instance, in algebraic topology, algebraic $K$-theory, and derived algebraic geometry. Such structured ring spectra or "geometric rings" are most simply viewed as algebraic-topological generalizations of the notion of ring from algebra and algebraic geometry. This talk will describe recent progress, in joint work with M. Ching, on an analog of Quillen–Sullivan theory for structured ring spectra.

**Kathryn Hess (EPFL)**
**The Boardman-Vogt tensor product of operadic bimodules**

Abstract: The Boardman-Vogt tensor product of operads endows the category of operads with a symmetric monoidal structure that codifies interchanging algebraic structures. In this talk I will explain how to lift the Boardman-Vogt tensor product to the category of composition bimodules over operads. I will also sketch two geometric applications of the lifted B-V tensor product, to building models for spaces of long links and for configuration spaces in product manifolds. This is joint work with Bill Dwyer.

**Michele Intermont (Kalamazoo College)**
**Some Results in Topological Clustering: Persistence, Stability, and "Mapper"**

Abstract: In this talk, we'll discuss some stability results for the topological clustering tool, Mapper, introduced by Singh, Memoli and Carlsson in 2007. We'll mention some preliminary work in using Mapper to analyze some fMRI data.

**Kristen Mazur (Lafayette College)**
**Additional Structure on the Category of Mackey Functors**

Abstract: The stable homotopy groups of a $G$-spectrum are Mackey functors, and the zeroeth stable homotopy group of a commutative $G$-ring spectrum has the extra structure of a Tambara functor. However, while Mackey functors and Tambara functors make frequent appearances in equivariant stable homotopy theory, much of their underlying algebra remains mysterious. I will discuss a new structure on the category of Mackey functors such that Tambara functors are commutative algebra-like objects. Moreover, the advantage to this new structure is that it is concrete and computable.

**John Peter (Utica College)**
**Charged Spaces**

Abstract: Let $C$ be a model category with an initial object $\Box$ and functorial factorizations. Let $S : C \to C$ be the suspension functor. An object $X$ of $C$ is said to be charged if it comes equipped with a map $S\Box \to X$. If $Y$ is any object of $C$, then $SY$ has a preferred charge, given by applying suspension to the map $\Box \to Y$. This motivates the question of whether a given charged object is a suspension up to a weak equivalence in a way that preserves charge structures. We study this question in the context of spaces over a given space, where we give a complete obstruction in a certain metastable range. As an application we show how this can be used to study when an embedding into a smooth manifold of the form $N \times I$ compresses to an embedding into $N$.

**Vesna Stojanoska (MIT)**
**Fixed points for unitary group actions on orthogonal decomposition spaces**

Abstract: For a fixed integer $n$, consider the nerve $\mathcal{L}_n$ of the topological poset of orthogonal decompositions of complex $n$-space into proper orthogonal subspaces. This space $\mathcal{L}_n$ has an action by the unitary group $U(n)$, and we study the fixed points for subgroups of $U(n)$. Given a prime $p$, we determine the relatively small class of $p$-toral subgroups of $U(n)$ which have non-empty fixed points. Note that $p$-toral groups are a Lie analogue of finite $p$-groups, thus if we are interested in the $U(n)$-space $\mathcal{L}_n$ at a fixed prime $p$, only the $p$-toral subgroups of $U(n)$ play a significant role. The space $\mathcal{L}_n$ is strongly related to the $K$-theory analogues of the symmetric powers of spheres and the Weiss tower for the functor that assigns to a vector space $V$ the classifying space $BU(V)$.Our results are a step toward a $K$-theory analogue of the Whitehead conjecture as part of the program of Arone-Dwyer-Lesh.This is joint work with J. Bergner, R. Joachimi, K. Lesh, K. Wickelgren.

**Marco Varisco (University at Albany, SUNY)**
**On the Adams Isomorphism for Equivariant Orthogonal Spectra**

Abstract: In joint work with Holger Reich we give a natural construction and a direct proof of the Adams isomorphism for equivariant orthogonal spectra. More precisely, for any finite group $G$, any normal subgroup $N$ of $G$, and any orthogonal $G$-spectrum $X$, we construct a natural map $A$ of orthogonal $G/N$-spectra from the homotopy $N$-orbits of $X$ to the derived $N$-fixed points of $X$, and we show that $A$ is a stable weak equivalence if $X$ is $N$-free. This recovers a theorem of Lewis, May, and Steinberger in the equivariant stable homotopy category, which in the case of suspension spectra was originally proved by Adams. We emphasize that our "Adams map" $A$ is natural even before passing to the homotopy category. One of the main tools we develop is a fibrant replacement construction with good functorial properties, which we believe is of independent interest.

**David White (Wesleyan University)**
**Motivic localization, cellularization, and Brown-Comenetz duality**

Abstract: For classical homology localization on spectra, localizing an additive homology theory $E$ can be done in two equivalent ways: one way is to localize the maps $f$ such that $E_*(f)$ is an isomorphism, the other is to localize the maps $f$ such that $f \wedge E$ is a weak equivalence. Motivically, these are no longer equivalent. The latter has been proven to exist for all motivic homology theories $E$. In this talk I will discuss recent work, joint with Carles Casacuberta, which proves that the former type of localization (now referred to as $E_{*,*}$-localization) exists. Furthermore, taking $E$ to be the sphere spectrum $S$ we can prove that the image of this localization functor is exactly the subcategory of cellular motivic spectra. This subcategory is the localizing subcategory generated by the spheres, by definition. We introduce the motivic analog of Brown-Comenetz duality and then prove that the subcategory above is cogenerated as a colocalizing subcategory by the motivic Brown-Comenetz dual of the sphere spectrum.