

Union College Mathematics Conference

December 3-4, 2005

Schedule of speakers

		Invited addresses Bailey 201	Session I Bailey 106	Session II Bailey 104	Session III Bailey 100
Saturday	9:30-10:00		R. Cockett	D. Yau	I. Soprunov
	10:10-10:40		P. Hofstra	M. Behrens	M. Huibregtse
	10:50-11:20		R. Blute	H. Park	P. Monsky
	11:30-12:30	D. Cox			
	2:00-2:30		J. Egger	A. Mauer-Oats	A. Pacelli
	2:40-3:10		M. Jackson	I. Volic	A. Milas
	3:20-3:50		P. Freyd	J. Armstrong	
	4:30-5:30	J. Grodal			
Sunday	9:30-10:30	J. Baez			
	10:50-11:20		E. Cheng	M. Johnson	J. Mermin
	11:30-12:00		N. Gurski	A. Salch	A. Chernev
	1:30-2:00		J. Morton	V. Chernov	L. Rose
	2:10-2:40		N. Yanofsky		S. Chaiken
	2:50-3:20		R. McGrail	G. Friedman	A. Martsinkovsky

Coffee and donuts will be available in Bailey 204 from 8:30 to 9:30 each morning. There will also be a coffee break on Saturday afternoon from 3:50-4:30, between the parallel sessions and the invited address, in Bailey 204.

Social Events

Friday 8:00-10:00 p.m. Reception and registration in Bailey 204

Saturday 6:30 p.m. Banquet in Hale House

Saturday 8:00 p.m. Party in Everest Lounge

Abstracts for Plenary Speakers

John Baez (University of California, Riverside)

“Higher Gauge Theory: 2-Connections”

Abstract: Gauge theory describes the parallel transport of point particles using the formalism of connections on bundles. In both string theory and loop quantum gravity, point particles are replaced by 1-dimensional extended objects: paths or loops in space. This suggests that we seek some kind of “higher gauge theory” that describes the parallel transport as we move a path through space, tracing out a surface. To find the right mathematical language for this, it seems we must “categorify” concepts from differential geometry, replacing smooth manifolds by smooth categories, Lie groups by Lie 2-groups, Lie algebras by Lie 2-algebras, bundles by 2-bundles, sheaves by stacks or gerbes, and so on. We give an overview of higher gauge theory, with an emphasis on the concept of “2-connection” for a principal 2-bundle. (Slides are available at <http://math.ucr.edu/home/baez/union/>)

David Cox (Amherst College)

“Two stories about implicitization and commutative algebra”

Abstract: This talk will discuss some of the unexpected relations between geometric modeling and commutative algebra. I will discuss moving lines and moving planes. These give interesting ways of representing parametric curves and surfaces in the plane and 3-dimensional space, respectively, and are closely related to syzygy modules in commutative algebra. The story for curves will involve free resolutions and the Hilbert-Burch theorem, while the surface case has relations to the Boeing 777, the Guggenheim Bilbao, resultants, local complete intersections, and the Serre conjecture.

Jesper Grodal (University of Chicago)

“Homotopical group theory”

Abstract: To a group one can associate its classifying space. This allows one to apply the toolkit of algebraic topology to groups. One can for instance localize a group at a prime, or ask if it can be suitably glued together out of smaller groups. One can also ask if groups can exist at just one prime, and to which extend such p-local objects glue together to give global groups? My talk will be an introduction to this kind of “homotopical group theory.”

Abstracts for Parallel Sessions

John Armstrong (Yale University)

“The Extension of Knot Groups to Tangles”

Abstract: The extension of the knot group $\pi_1(S^3 \setminus K)$ to the category of tangles is introduced via a new category-theoretic construction. Through this presentation, a new avenue of proof for results about knot groups is opened.

Mark Behrens (MIT)

“Hypercohomology of categories”

Abstract: How do you combine sheaf cohomology with the cohomology of a category? I will discuss enriched categories where the objects, as well as the morphisms, are enriched. What form does a coefficient system (enriched functor) take for this set-up? This talk is largely expository, combining a definition in old paper of Morava’s with some modern terminology.

Rick Blute (University of Ottawa)

“Deep Inference and Girard’s Probabilistic Coherence Spaces”

Abstract: (Joint work with Prakash Panangaden and Sergey Slavnov) We propose a definition of categorical model of the deep inference system BV. Deep inference systems are an extension of traditional sequent calculus which have a strong symmetry between premises and conclusions, and allow substitutions of arbitrary depth within a proof. The specific system BV is an extension of multiplicative linear logic to include Retore’s noncommutative, self-dual connective.

Our definition is based on the Cockett-Seely notion of linear functor. Basically we assume a *-autonomous category with a bivariate linear functor, satisfying a degeneracy condition.

We will also show that Girard’s category of probabilistic coherence spaces is an example and discuss possible applications to discrete quantum causal dynamics.

Seth Chaiken (University at Albany)

“Ported alias Set-Pointed and Non-scalar Tutte Functions”

Abstract: A Tutte function or invariant traditionally maps matroids into a commutative ring. After a brief exposition in terms of support sets (or signed sets) of syzygies when the matroid is representable, we use the setting where the deletion/contraction identities that characterize Tutte functions are restricted to elements not in a distinguished subset we call ports. The resulting universal Tutte-like polynomials include variables that signify connected unoriented or oriented matroids on subsets of ports. We survey new and old identities, multilinear operations and transformations on such polynomials that correspond to various operations on, and combinations of, unoriented or oriented matroids. Some of the identities correspond to identities on particular kinds of representations of the matroids. For example, the ported Tutte equations interpreted for a certain ported matroid-to-matroid function correspond to a non-commutative variant of the ported Tutte equations interpreted for matroid realizations expressed in exterior algebra. Other examples involve ported matroid union, its dual, and generalized parallel connection. The topic is related to resistive electrical network or random walk models with their Laplacian determinants, and to splitting formulas for Tutte polynomial computations on decomposed graphs.

Eugenia Cheng (University of Chicago)

“Towards an n -category of cobordisms”

Abstract: (joint work with Nick Gurski) We discuss an approach to constructing a weak n -category of cobordisms. First we present a generalisation of Trimble’s definition of n -category which seems most appropriate for this construction; in this definition composition is parametrised by a contractible operad. Then we discuss the problem of defining an n -category **nCob**, whose k -cells are k -cobordisms, possibly with corners. We show how to make some preliminary constructions as “stepping stones” towards the desired goal. We follow Baez and Langford in using “manifolds embedded in cubes” rather than general manifolds. As a yet further preliminary to that, we show how to put a Trimble n -category structure on general subsets of n -cubes.

Vladimir Chernov (Dartmouth College)

“New algebraic structures on the generalized string algebra”

Abstract: A garland based on a manifold N is a few copies of N that are glued together at marked points. For a fixed manifold M let \mathcal{N} be the space of mappings into M of garlands based on N . Rudyak and myself showed that the bordism group $\Omega_*(\mathcal{N})$ of \mathcal{N} has a structure of a Lie algebra that is related to Chas-Sullivan string algebra. In the talk we show that this Lie bracket is the generalization of Goldman Lie bracket of free loops on a surface. We also discuss the relation of the Lie-bracket to the Hatcher-Quinn invariant and to Vassiliev invariants. Finally we introduce a new Lie algebra structure on $\Omega_*(\mathcal{N})$ and explain how it is related to fixed point and coincidence theory.

Robin Cockett (University of Calgary)**“Linking functors and linking envelopes”**

Abstract: Linking functors are rather special lax functors between restriction categories: they are for restriction categories what “dual pre-homorphism” are to inverse semigroups. They seem to play a central role in the morphisms which define atlases for generalized manifolds (e.g. orbifolds). It is natural to wonder whether these morphisms can be turned into honest restriction functors and, in fact, it turns out they can. Formally there is a left adjoint to the underlying functor from restriction categories with restriction functors to the category of restriction functors with linking functors: the adjoint is given by the “linking envelope”. Furthermore this construction can be extended to join restriction categories. The former construction, and its universal property, has been noticed for inverse semigroups: Lawson, Margolis, Steinberg (2002). The extension of the result for joins I have not been able to locate.

Jeff Egger (Dalhousie University)**“Of operator algebras and operator spaces”**

Abstract: One of the recent advances in Functional Analysis has been the introduction of the notion of an (abstract) operator space. This can be seen as a refinement of the notion of a Banach space which (among other things) solves the problem that not every Banach algebra is an operator algebra. Which theorems about Banach spaces generalise to operator spaces? This question would be easier to answer if one could prove Pestov’s Conjecture: that there exists a Grothendieck topos whose internal Banach spaces are equivalent to operator spaces. I will report on progress towards proving Pestov’s conjecture.

Peter Freyd (University of Pennsylvania)**“Homological Algebra Revisited”**

Abstract: What appears to be never-before-noticed *-autonomous structures on certain familiar functor categories sheds new light on classical homological algebra.

Greg Friedman (Vanderbilt University/TCU)**“Singular chains in intersection homology”**

Abstract: We will provide an introduction to intersection homology theory, which is a generalization of homology theory particularly well-suited to the study of manifold stratified spaces (including algebraic varieties). We will also discuss our recent results that bridge the gap between chain theoretic and sheaf theoretic versions of intersection homology

Nick Gurski (University of Chicago)**“Nerves of bicategories”**

Abstract: In this talk I will discuss the simplicial nerve of a bicategory and theorems characterizing the simplicial sets arising as these nerves. Two such theorems will be discussed, one old and one new. I will also explain the connection with Street’s definition of weak ω -category and give an indication of how a definition akin to Street’s but in finite dimensions should be developed.

Pieter Hofstra (University of Calgary)

“The geometry of realizability”

Abstract: Partial Combinatory Algebras (PCAs) and their generalizations form the basic building blocks for realizability toposes, just as Grothendieck toposes can be obtained from sites. In fact, the 2-category of realizability toposes is equivalent to a certain 2-category of PCAs, and it is because of this fact that one would like to get a better understanding of the latter category. In this talk, I will discuss some 2-categorical aspects of the 2- category of PCAs. In particular, we will look at a pseudo-factorization system, which is induced by a version of the comma construction. I will also discuss a connection with Pitts’ iteration theorem.

Mark Huibregtse (Skidmore College)

“An elementary construction of the Hilbert scheme of points of an affine space”

Abstract: Let k be an algebraically closed field, and let $k[\mathbf{x}]$ denote the polynomial ring $k[x_1, \dots, x_n]$. We describe an elementary construction of $\text{Hilb}_{A_k^n}^d$, the Hilbert scheme of d points of the affine space $A_k^n = \text{Spec}(k[\mathbf{x}])$. The k -points of $\text{Hilb}_{A_k^n}^d$ correspond to the closed subschemes of A_k^n that are defined by ideals $I \subseteq k[\mathbf{x}]$ such that $k[\mathbf{x}]/I$ is k -free of dimension d , the **colength** of I . $\text{Hilb}_{A_k^n}^d$ is covered by a finite collection of affine open subschemes U_β , where β denotes a set of d monomials in x_1, \dots, x_n having the property that if $m \in \beta$ and $m_1 \mid m$, then $m_1 \in \beta$; as a set of k -points, U_β consists of the points corresponding to ideals I such that the quotient $k[\mathbf{x}]/I$ is k -free with basis β . The subscheme U_β represents a sub-functor of the Hilbert functor represented by $\text{Hilb}_{A_k^n}^d$, and the existence of U_β (as an object representing this sub-functor) can be established using only polynomial arithmetic, nothing more advanced (such as Grassmannians). One obtains a concrete description of the coordinate ring of U_β as a quotient of a polynomial ring. $\text{Hilb}_{A_k^n}^d$ is then constructed by gluing the U_β together along overlaps.

Matthew Jackson (University of Pittsburgh)

“Translating the elements of Measure Theory into Topos Theory”

Abstract: In 1969, Dana Scott showed that in the topos of sheaves on a σ -algebra, the object of Dedekind real numbers is the sheaf of measurable real valued functions. In 1977 Sigfried Breitsprecher showed that the measures on a σ -algebra formed a sheaf \mathbb{M} .

These results suggest that it is possible to translate measure theory into a topos theoretic framework. However, in order to do this, we would need an internal description of \mathbb{M} . In this talk, a logical formula will be presented that characterizes \mathbb{M} . A morphism capturing Lebesgue integration will also be presented.

Mark Johnson (Penn State Altoona)

“Realizing Diagrams of Pi-algebras ”

Abstract: In joint work with David Blanc and Jim Turner, we have constructed an obstruction theory for realizations of diagrams in terms of Andre-Quillen cohomology. In certain cases we can also identify the cohomology of a diagram in terms of that of its component spaces.

Alex Martsinkovsky (Northeastern University)

“On the Torsion Problem of Reiffen and Vetter”

Abstract: In the lecture I will give a simple criterion for the torsion submodule of a finitely generated module to contain minimal generators of the module and will describe the origins of this problem. All terms will be defined and explained.

Andrew Mauer-Oats (Northwestern University)

“**Assembly maps for iterated cross effects**”

Abstract: We give a geometric construction related to the Whitehead product that creates assembly maps for cross effects of the identity

$$cr_k(cr_{a_1}, \dots, cr_{a_k}) \rightarrow cr_{\Sigma a_i}.$$

After applying a suitable multilinearization process, these maps should become structure maps for an operad on the derivatives. This process should generalize to the cross effects of monads other than the identity.

Robert McGrail (Bard College)

“**Slice Categories and the Continuum Hypothesis**”

Abstract: The slice construction is a means to freely attach a generic element to a category with all finite limits. For toposes, this translates to freely adjoining generic subobjects. This talk discusses the relationship between a particular application of the slice construction and models of ZFC that violate the continuum hypothesis. This is work in progress and my main conclusion is probably known to set theorists. However, this presentation makes the result clear to a categorist who would rather not dig into the details of model theory and axiomatic set theory.

Jeff Mermin (Cornell University)

“**Ideals containing the squares of the variables**”

Abstract: We study the Betti numbers of graded ideals containing the squares of the variables, in a polynomial ring. We prove the lex-plus-powers conjecture for such ideals: Let k have characteristic 0 and $S = k[x_1, \dots, x_n]$. Set $P = (x_1^2, \dots, x_n^2)$. Let I be any graded ideal containing P . By Kruskal-Katona’s theorem, there is a lexicographic ideal L such that $L + P$ has the same Hilbert function as I . We show that the graded Betti numbers of $L + P$ are greater than or equal to those of I . (Joint with Irena Peeva and Mike Stillman.)

Antun Milas (University at Albany)

“**Representations of N=1 vertex operator superalgebras**”

Abstract: N=1 vertex operator superalgebras are vertex superalgebras equipped with action of the N=1 Neveu-Schwarz Lie superalgebra. We will discuss the recent progress made in constructing modular tensor categories from N=1 vertex operator superalgebras associated to N=1 superconformal minimal models.

Paul Monsky (Brandeis University)

“**Hilbert-Kunz theory for plane curves**”

Abstract: I’ll present results of Brenner, Trivedi, Teixeira and me, and make some guesses based on computer calculations as to what might be true for singular plane curves.

Jeffrey Morton (University of California, Riverside)
“Categorifying the Quantum Harmonic Oscillator”

Abstract: Finding a combinatorial model for some mathematical entity is an example of the process called “categorification”. This involves the interpretation of $\mathbb{N}[x]$ as the Burnside rig of the category of combinatorial species. There are surprising applications to quantum mechanics, and in particular the quantum harmonic oscillator, via Joyal’s “species”, a new generalization called “stuff types”, and operators between them. These “stuff operators” can be represented as Feynman diagrams for the oscillator. These are most properly described in terms of a slice 2-category. We will show how to construct a combinatorial model for the quantum harmonic oscillator in which the group of phases, $U(1)$, plays a special role. We describe a general notion of “ M -Stuff Types” for any monoid M , and see that the case $M = U(1)$ provides an interpretation of time evolution in the combinatorial setting, as well as other quantum mechanical features of the harmonic oscillator.

Allison Pacelli (Williams College)
“High n -Rank in Class Groups of Global Fields”

Abstract: Determining the structure of the class group of a number field or function field is a long standing problem in algebraic number theory. It is known that for any integers m and n , there are infinitely many number fields and function fields of fixed degree m with class number divisible by n . A natural generalization is to consider the n -rank of the class group. In this talk, we’ll show that the n -rank of the class group of a function field is closely related to the unit rank. We’ll also look at the special case of 3-rank in quadratic number fields.

HeeSook Park (Vassar College)
“The dimensions of the second bounded cohomology of perfect groups”

Abstract: K. Fujiwara conjectured that the second bounded cohomology of a group is zero or infinite dimensional as a vector space over \mathbb{R} . However, it is known that there are some simple groups for which the second bounded cohomology is not zero but finite dimensional. We investigate the dimensions of the second bounded cohomology and its singular part of perfect groups as a vector space over \mathbb{R} .

Lauren Rose (Bard College)
“Piecewise polynomials with boundary conditions”

Abstract: We will first describe the algebraic structure of modules of piecewise polynomials over polynomial rings in several variables. Then we will discuss what happens to the modules when you add boundary conditions to the domain. The latter part of the talk is work in progress.

Andrew Salch (University of Rochester)
“Number-theoretic approaches to the Morava stabilizer group”

Abstract: For a choice of a positive integer h and a prime number p , there is a pro- p -group called the Morava stabilizer group which carries important information about the $K(h)$ -local sphere at the prime p . In particular, the cohomology of the Morava stabilizer group, with appropriate coefficients, is the E_2 term of a spectral sequence converging to $\pi_*(L_{K(h)}S)$. Unfortunately, the cohomology of the Morava stabilizer group is known for only a handful of values of h and p . The most effective computational technique known today is a combination of a certain equivariant May spectral sequence and a number of restriction maps originally studied by Hans-Werner Henn; in this talk, we will review basic facts about the Morava stabilizer group and then define “generalized stabilizer groups” which let us describe Henn’s maps using local class field theory, and give us a better foothold on the problem of computing the cohomology of these groups. Time allowing, we will also demonstrate some new computations that these techniques have allowed.

Ivan Soprunov (University of Massachusetts, Amherst)

“Global residues for sparse polynomial systems”

Abstract: The global residue is a fundamental invariant of multivariate polynomial systems with finitely many roots. It is a linear function that maps every polynomial to a complex number which depends rationally on the coefficients of the system. The global residue goes back to the works of Euler, Jacobi and Kroneker. In the toric setting the global residue was studied by Khovanskii. We will present a new algorithm for computing the global residue explicitly for any given sparse polynomial system whose Newton polytopes are full-dimensional. This generalizes the previously known result by Cattani and Dickenstein when the polytopes have the same normal fan. Our result gives rise to interesting questions about lattice points in the Minkowski sum of polytopes.

Alex Tchernev (University at Albany)

“On the Betti numbers of multigraded modules”

Abstract: Let M be a multigraded module over a polynomial ring R . In the case when M is cyclic (i.e. of the form R/I for some monomial ideal I) there are formulas due to Hochster that express the Betti numbers of M in terms of the reduced homology of certain simplicial complexes associated with the monomial generators of I . No such formulas are known for general multigraded modules. In this talk we will discuss what can be said on this question when the module M is of what we call a “generic type”. In that situation it is possible to associate with M a collection of simplicial complexes, and compute the Betti numbers of M from the reduced homology of these complexes. This is joint work with Hara Charalambous.

Ismar Volic (University of Virginia)

“Calculus of functors, operad formality, and embedding spaces”

Abstract: We use calculus of functors and formality of the little discs operad to study the rational homotopy type of certain spaces of embeddings of a manifold in a vector space. In particular we deduce the collapse of some spectral sequences converging to the rational homology and homotopy of those spaces of embeddings. One of the consequences is a complete description of the rational homotopy type of spaces of knots in high codimension. This is joint work with Greg Arone and Pascal Lambrechts.

Noson Yanofsky (Brooklyn College)

“Towards a Definition of an Algorithm”

Abstract: We look at an algorithm as an equivalence class of programs. Two programs are equivalent if they are “essentially” the same program. In order to explore these ideas, we look at the set of primitive recursive functions. Each primitive recursive function can be described by labeled binary trees that show how the function is built up. In a sense, each tree is like a program that describes how to compute a function. We give relations that say when two such trees are “essentially” the same. An equivalence class of such trees will be called an algorithm. We give a complexity measure for such trees and show that if two descriptions are essentially the same, then their complexity is the same. Further complexity issues are explored.

Donald Yau (Ohio State University)

“Gerstenhaber algebra structure on dendriform cohomology”

Abstract: Dendriform algebras arise from Loday’s program of studying periodicity phenomenon in algebraic K-theory. I will talk about the existence of a G -algebra structure on dendriform algebra cohomology, which is analogous to the one on Hochschild cohomology of associative algebras.