Union College Mathematics Conference
November 8-9, 2003

Schedule of speakers

		Invited addresses Bailey 207	Session A Bailey 207	Session B Bailey 100	Session C Bailey 201
Saturday	9:30-10:00		G. Friedman		M. Tierney
	10:10-10:40		V. Chernov		S. Forcey
	10:50-11:20		I. Volic		M. Bunge
	11:30-12:30	A. Joyal			
	2:00-2:30		A. Mauer-Oats		D. Pronk
	2:40-3:10		A. Elmendorf	A. Hwang	J. Kennison
	3:20-3:50		M. Arkowitz	E. Dryden	M. Weber
	4:00-4:30		I. Johnson	V. Apostolov	P. Hofstra
	5:00-6:00	C. LeBrun			
Sunday	9:30-10:30	Ulrike Tillmann			
	10:50-11:20		K. Potocka	R. Foote	M. Barr
	11:30-12:00		O. Roendigs	J. Harnad	J. Funk
	1:20-1:50		A. Crans		R. Cockett
	2:00-2:30		F. Linton		R. Seely
	2:40-3:10				J. Egger

Coffee and donuts will be available in Bailey 204 from 8:30 to 9:30 each morning. There will also be a coffee break on Saturday afternoon from 4:30-5:00, between the parallel sessions and the invited address, in Bailey 204.

Social Events

Friday 8:00-10:00 p.m. Reception and registration in Bailey 204
Saturday 6:30 p.m. Dinner in Hale House
Saturday 8:30 p.m. Party at Kathryn Lesh's house, 1524 Baker Ave.

Abstracts for Plenary Speakers

André Joyal, Université du Québec à Montréal
"The Witt vectors construction revisited"
Abstract: Witt vectors are playing an important role in number theory and topology. Introduced by Witt in 1937, they were generalised by Cartier in 1967. But they remain mysterious objects with surprisingly many facets. We shall describe the Witt vector construction as a right adjoint to the forgetful functor from theta-rings to rings. Theta-rings were discovered by Bousfield in his work on the K-theory of H-spaces and independantly by the author. A characterisation of forgetful functors having a right adjoint was given by Wraith and Tall in 1970.

Claude LeBrun, SUNY Stony Brook
"The Curvature of 4-Manifolds"
Abstract: One of the basic aims of modern differential geometry is to relate the curvature of a Riemannian manifold to its topology. In this lecture, I will discuss several different notions of curvature, and some differential-topological invariants of smooth compact 4-manifolds which arise from them. Amazingly enough, it turns out that different smooth structures on a fixed topological 4-manifold can often be distinguished from one another in terms of the curvature properties of the Riemannian metrics they support.

Ulrike Tillmann, Oxford University
"The stable cohomology of mapping class groups - A homotopy approach"
Abstract: The cohomology of mapping class groups of surfaces (or moduli spaces of Riemann surfaces) has been studied intensively by different mathematical schools during the last 20 years. I will give a survey of recent results and the ideas behind them for the stable mapping class group, in particular the proof of Mumford's conjecture by Madsen and Weiss.

Abstracts for Parallel Sessions

Vestislav Apostolov, Université du Québec à Montréal
"Extremal Kähler metrics and hamiltonian 2-form"
Abstract: We will introduce the notion of a hamiltonian 2-form and explain how this is related to a number of explicit constructions and classification results in Kähler Geometry. We will discuss applications to toric orbifolds as well. The talk is based on a joint work with David Calderbank, Paul Gauduchon and Christina Tønnesen-Friedman.

Martin Arkowitz, Dartmouth College
"The Sectional Category of a Map"
Abstract: A generalization of the Svarc genus of a fiber map is studied. For an arbitrary collection $\mathcal{E}$ of spaces and a map $f : X \to Y$, we define a numerical invariant, the $\mathcal{E}$-sectional category of f, in terms of open covers of Y. We obtain several basic properties of $\mathcal{E}$-sectional category, including those dealing with homotopy domination and homotopy pushouts. We give three simple axioms which characterize the $\mathcal{E}$-sectional category. Finally we study the following questions: (1) How is the $\mathcal{E}$-sectional category of f related to the domain and the target of f? (2) How does $\mathcal{E}$-sectional category behave with respect to composition of maps? (3) What is the relation between the $\mathcal{E}$-sectional category, the Clapp-Puppe category and the Fadell-Husseini category?

Joint with Jeffrey Strom.

Michael Barr, McGill University
"On CR-epic maps and absolute CR-epic spaces"
Abstract: A map $X \to Y$ of topological spaces (all spaces are completely regular Hausdorff) is called CR-epic if the induced map $C(Y) \to C(X)$ is an epimorphism in the category CR of commutative rings. A space is an absolute CR-epic if every embedding into any other space is CR-epic. It is sufficient if every dense embedding into a compact space is CR-epic. For example, any space that is locally compact and σ-compact is absolute CR-epic. This talk will also discuss some other examples.

This is joint work with Robert Raphael and Grant Woods.

Marta Bunge, McGill University
"Groupoids, locally paths simply connected toposes, and the van Kampen theorems"
Abstract: The use of groupoids in the van Kampen theorems (for spaces) has been advocated by Brown (1966) and Grothendieck (1982). In topos theory, groupoids appear naturally in the construction of Moerdijk and Wraith (1986) of the paths fundamental groupoid $\Pi_1(\mathcal{E})$ of an unpointed topos $\mathcal{E}$. The (paths groupoids version of the) van Kampen theorem for toposes would roughly say that, if the pseudofunctor Π_1 is applied to certain pushout squares of toposes, then it gives pushout squares of groupoid toposes. An obvious strategy would then be to try to show that under the conditions imposed, the pseudofunctor Π_1 is a left adjoint, hence preserves (all) pushouts. We give two versions here. The first version of the van Kampen theorem in terms of Π_1 that we give here is a direct consequence of putting together results of Bunge-Lack (2003) and of Bunge-Moerdijk, under the assumption of type (1) that all toposes $\mathcal{E}$ involved are locally simply connected and have paths simply connected universal covers. The second version of the van Kampen theorem in terms of Π_1 that we give here results from examining under which conditions the construction of Π_1 is a totally disconnected reflection (in the sense naturally associated with the unit interval I as an exponentiable locale or topos), hence a left adjoint. In this case, we find that a suitable assmption to make is (2) that the evaluation at the endpoints $\varepsilon\colon \mathcal{E}^I \to \mathcal{E} \times \mathcal{E}$ is locally connected. Assumptions (1) and (2), in turn, both follow from assuming that $\mathcal{E}$ is locally paths simply connected.

Vladimir Chernov, Dartmouth College
"Affine linking numbers, their applications and algebraic structures on generalized strings and the general theory of Affine linking numbers."
Abstract: The linking number is a classical invariant of two zero- homologous submanifolds $N_1, N_2 \subset M$. We construct the theory of "Affine linking invariants" that are natural generalizations of linking numbers to the case of non-zerohomologous N_1 and N_2. We show that affine linking numbers allow one to make conclusions about causality relations between events from the picture of wave fronts of the events at a certain time moment without any knowledge of the front propagation law. They also allow to calculate the algebraic number of times a wave front has passed through a given point between two time moments. A garland based on a manifold P is a finite set of manifolds homeomorphic to P with some of them glued together at marked integer graded points. The construction of the "Affine linking numbers"" is based on our generalization of the Chas-Sullivan string homology of the space of mappings $S^1 \to M$, to the case of bordism groups of the space of mappings of garlands based on an arbitrary manifold P. (In the construction of Chas-Sullivan string homology it was essential that S^1 is a co-H-space, and homotopy spheres are the only closed manifolds with a co-H-space structure.) We show that Lie super algebra, associative algebra, and other algebraic structures of the string homology of Chas-Sullivan admit generalizations to the case of bordism groups of mappings of garlands based on an arbitrary manifold P.

Joint work with Yuli B. Rudyak.

Robin Cockett, University of Calgary
"Introduction to range restriction categories"
Abstract: Restriction categories provide a compact equational axiomatization for categories of partial maps. They come equipped with a completeness theorem which asserts that every restriction category is a full subcategory of a real category of partial maps.

Often, however, one would wishes for the subobjects over which the partiality is defined to be part of a (stable) factorization system. It turns out that this also has a very simple equational axiomatization for which there is a similar completeness theorem. These categories are called range restriction categories.

Given a category, from general principles, one now knows that it is possible to generate the free range restriction category associated with it. Even more concretely, one can therefore generate the free range restriction category generated from a graph ... and I shall describe this construction.

This is joint work with Xiuzhan Guo.

Alissa S. Crans, University of California, Riverside
"Lie 2-algebras"
Abstract: The theory of Lie algebras can be categorified starting from a new notion of '2-vector space', which we define as an internal category in Vect. We define a 'semistrict Lie 2-algebra' to be a 2-vector space L equipped with a skew-symmetric bilinear functor $[\cdot,\cdot] : L \times L \to L$ satisfying the Jacobi identity up to a linear natural transformation called the 'Jacobiator', which in turn must satisfy a certain law of its own. This law is closely related to the Zamolodchikov tetrahedron equation, and indeed we prove that any semistrict Lie 2-algebra gives a solution of this equation, just as any Lie algebra gives a solution of the Yang–Baxter equation.

Emily Dryden, Dartmouth College
"Inverse Spectral Problems on Riemannian Orbifolds"
Abstract: Let M be a Riemannian manifold, and let $\Delta = -$ div grad be the Laplace operator on smooth functions on M. The focus of inverse spectral theory is to determine how much geometric information about M is encoded in the eigenvalue spectrum of Δ. We generalize several inverse spectral results on manifolds to the setting of Riemannian orbifolds. For example, in analogy with a result of H.P. McKean for compact Riemann surfaces, we show that sets of isospectral compact hyperbolic two-dimensional orbifolds have finite cardinality.

Jeff Egger, University of Ottawa
"Categories of Adherence Spaces"
Abstract: "Adherence space" is a term I have come to apply to objects of many different, but related, monoidal bi-closed categories. I will present definitions and address the central question of when a category of adherence spaces is $*$-autonomous.

Anthony D. Elmendorf, Purdue University Calumet
"Multicategories in K-theory and stable homotopy"
Abstract: A multicategory is a simultaneous generalization of an operad and a symmetric monoidal category. It can be thought of as an "operad with many objects" in exactly the same way a category can be thought of as a "monoid with many objects." This talk will discuss joint work with Mike Mandell in which we make two uses of multicategories: as a framework for theorems about multiplicative structure in algebraic K-theory, and as a objects in their own right, forming a "completion" of the category of small symmetric monoidal categories, with K-theory of their own.

Robert L. Foote, Wabash College
"**Integral-Geometric Formulas for Perimeter in $\mathbf{S}^2$, $\mathbf{H}^2$, and Hilbert Planes**"
Abstract: We develop two types of integral formulas for the perimeter of a convex body K in planar geometries. We derive Cauchy-type formulas for perimeter in planar Hilbert geometries. Specializing to $\mathbf{H}^2$ we get a formula that appears to be new. In the projective model of $\mathbf{H}^2$ we have $\mathcal{P} = \frac{1}{2}\int w\,d\varphi$. Here w is the Euclidean length of the projection of K from the ideal boundary point $R = (\cos\varphi, \sin\varphi)$ onto the diametric line perpendicular to the radial line to R (the image of K may contain points outside the model). We show that the standard Cauchy formula $\mathcal{P} = \int \sinh r\,d\omega$ in $\mathbf{H}^2$ follows, where ω is a central angle perpendicular to a support line and r is the distance to the support line. The Minkowski formula $\mathcal{P} = \int \kappa_g \rho^2\,d\theta$ in $\mathbf{E}^2$ generalizes to $\mathcal{P} = \frac{1}{4\pi^2}\int \kappa_g L(\rho)^2\,d\theta + \frac{k}{2\pi}\int A(\rho)\,ds$ in $\mathbf{H}^2$ and $\mathbf{S}^2$. Here (ρ,θ) and κ_g are, respectively, the polar coordinates and geodesic curvature of ∂K, k is the (constant) curvature of the plane, and $L(\rho)$ and $A(\rho)$ are, respectively, the perimeter and area of the disk of radius ρ. In $\mathbf{E}^2$ this is locally equivalent to the Cauchy formula $\mathcal{P} = \int r\,d\omega$ in the sense that the integrands are pointwise equal. In contrast, the corresponding Minkowski and Cauchy formulas are not locally equivalent in $\mathbf{H}^2$ and $\mathbf{S}^2$.

Joint work with Ralph Alexander and I. D. Berg.

Stefan Forcey, Virginia Tech
"**Loop spaces, enrichment, and n–categories**"
Abstract: It has been shown that k–fold monoidal categories correspond to iterated loop spaces. I will discuss how enriching over these relates to delooping. The example of this is G–torsors. The iteration of enrichment extends the topological process and also suggests a description of weak n–categories that uses associahedra.

Greg Friedman, Yale University
"**Cobordism of disk knots**"
Abstract: We study cobordisms and cobordisms rel boundary of PL locally-flat disk knots $D^{n-2} \hookrightarrow D^n$. Cobordisms of disk knots without fixed boundaries are easily classified by the cobordism properties of their boundary sphere knots, and any two even-dimensional disk knots with isotopic boundary knots are cobordant rel boundary. However, the cobordism rel boundary theory of odd-dimensional disk knots is more subtle. Generalizing results of Levine on cobordism of sphere knots, we define disk knot Seifert matrices and show that two higher-dimensional disk knots with isotopic boundaries are cobordant rel boundary if and only if their disk knot Seifert matrices are algebraically cobordant. We also find necessary and sufficient conditions to realize a Seifert matrix cobordism class among the disk knots corresponding to a fixed boundary knot, assuming the boundary knot has no middle-dimensional 2-torsion. This classification is performed by relating the Seifert matrix of a disk knot to its Blanchfield pairing and by establishing a close connection between this Blanchfield pairing and the Farber-Levine torsion pairing of the boundary knot (in fact, for disk knots satisfying certain connectivity assumptions, the disk knot Blanchfield pairing will determine the boundary Farber-Levine pairing). We also study the dependence of disk knot Seifert matrices on choices of Seifert surface, demonstrating that all such Seifert matrices are rationally S-equivalent, but not necessarily integrally S-equivalent.

Jonathon Funk, University of Regina
"**Elementary Completeness**"
Abstract: A completeness condition for maps first formulated in the topological case by Ralph Fox can be expressed in terms of Grothendieck topologies (in a small category). Thus, we may generalize Fox's condition to geometric morphisms between toposes. In turn, this leads to an elementary formulation of the completeness condition in the sense that it is formulated in terms of the primitives of elementary topos theory.

Joint with M. Bunge, T. Streicher, M. Jibladze.

John Harnad, Centre de recherches mathématiques, Univ. de Montreal
"Bihamiltonian structures, Mukai brackets and R-matrices"
Abstract: For the rational, elliptic and trigonometric R-matrices, we exhibit the links between three "levels" of Poisson spaces: (a) Finite-dimensional spaces of matrix-valued holomorphic functions on the complex line or an elliptic curve; (b) Spaces of spectral curves and sheaves supported on them; (c) Symmetric products of a surface. At each level, there is a linear space of compatible Poisson structures, and the maps relating them are Poisson. This leads in a natural way to Nijenhuis coordinates for these spaces. The spectral invariants at level (a) determine Hamiltonian systems which are integrable for each Poisson structure in the linear family, such that the Lagrangian leaves are the intersections of the symplectic leaves over the Poisson structures in the family. Specific examples include many of the well-known integrable systems.

Pieter Hofstra, University of Ottawa
"Realizability Toposes"
Abstract: The construction of a realizability topos out of a partial combinatory algebra (PCA) is well known. I will show how to make this construction functorial and give a simple characterization of geometric morphisms between realizability toposes in terms of morphisms of PCAs.

Andrew D. Hwang, Holy Cross
"Equivariant Embeddings of Surfaces "
Abstract: Embedding diagrams are a pedegogical tool from general relativity, in which a 1+1-dimensional slice of a spherically-symmetric static spacetime is isometrically embedded in a 2+1-dimensional Minkowski space. There is a remarkable duality between embedding diagrams and a certain symplectic description of surfaces of revolution in Euclidean 3-space. Consequences include a simple formula for the Gaussian curvature of a spacetime slice in suitable coordinates.

(Joint work with John T. Giblin)

Inga Johnson, University of Rochester
"On the degree 2 map for a sphere"
Abstract: Consider the two self-maps of ΩS^{2n+1} given by

(1)Ψ the H-space squaring map, and

(2)$\Omega[2]$, the loop of the degree two map.

A classical result of Adams implies these two maps are homotopic iff $2n+1 = 1, 3$, or 7. We have reason to conjecture that the two maps become homotopic after looping $2n$ times. A natural question to ask is, what is the smallest i such that $\Omega^{i-1}\Psi \simeq \Omega^i[2]$?

The purpose of this talk is to give lower bounds on i for these two maps to be homotopic. For example, we prove the following theorem.

Theorem [Cohen - J.]: Let $j - 4 \geq k \geq 1$.

If the maps $\Omega^{i-1}\Psi$ and $\Omega^i[2] : \Omega^i S^{2^j+2^k-1} \to \Omega^i S^{2^j+2^k-1}$ are homotopic, then $i > 2^j + 2^k - 2^{j-3}$.

We consider other odd spheres S^{2n+1} for $2n+1 \neq 2^j - 1$.

Some of the tools we use are a secondary operation of Brown and Peterson along with factorizations of the Steenrod Squares. This gives information about factoring the Whitehead product which leads to information about the two maps in question.

(Joint work with F. R. Cohen)

John Kennison, Clark University and McGill University
"**Cyclic and Primal Spectra of Boolean Flows**"
Abstract: By a *flow* or *discrete dynamical system* in a category, we mean an object X together with a map $t : X \to X$. If t is a flow in compact Hausdorff spaces, we are interested in describing cyclic patterns in the orbit $(x, t(x), ..., t^n(x), ...)$ of $x \in X$. By using symbolic dynamics, we can approximate (X, t) by a flow in Stone spaces, or, equivalently, in Boolean algebras. We define "cyclic"" flows in any reasonable category and construct a cyclic spectrum for Boolean flows. We relate our results to well-known examples such as the logistic flow on the unit interval.

Fred E. J. Linton, Wesleyan University
"**Shedding some Localic and Linguistic Light on the Tetralemma Conundrums**"
Abstract: Numerous authors over the centuries have puzzled over what has been called "the Buddhist paradigm of *catuskoti*." A classic example: the four statements, considered both mutually exclusive and jointly exhaustive,
(i) the *Tathagata* exists after death;
(ii) the *Tathagata* does not exist after death;
(iii) the *Tathagata* both does and does not exist after death;
(iv) the *Tathagata* neither does nor does not exist after death.
We would offer some linguistic *gedanken*-experiments illustrating everyday situations in which appropriate analogues to these four statement-forms are entirely plausible as mutually exclusive, or jointly exhaustive, alternatives; and we offer a framework, based on the logical paradigms of locale theory, illustrating how forms (iii) and (iv), in particular, need be neither contradictory, nor paradoxical, nor even mutually equivalent.

Andrew Mauer-Oats, Purdue University
"**Towards a small complex for n-excisive approximations**"
Abstract: Alan Robinson has constructed a relatively small bicomplex $\Xi(F)$ whose total complex is the stabilization of F evaluated at the unit. (Here F is a functor from finite pointed sets to k-modules; for example, the Loday functor that gives rise to Hochschild homology.) There is another, larger complex due to Johnson and McCarthy that also computes the stabilization of a functor F. Using their tools, we investigate how the original Robinson-Whitehouse complex CF served as a precursor to $\Xi(F)$, looking for a direct (that is, motivated and generalizable) understanding of how CF produces the stabilization of F. The talk will include an explanation of why we are interested in this project.

Katarzyna Potocka, Lehigh University
"**The number of summands in v_1-periodic homotopy groups of $SU(n)$**"
Abstract: In 1991 Davis showed that if p is an odd prime, $v_1^{-1}\pi_{2k}(SU(n);p)$ is cyclic, gave the formula for its order, and proved that $v_1^{-1}\pi_{2k-1}(SU(n);p)$ has the same order as the even group above for a matching value of k, but is not always cyclic. My talk is a summarty report on the paper, in which I determined the number of summands of $v_1^{-1}\pi_{2k-1}(SU(n);p)$ for all values of p, k, and n, where p is an odd prime. The method that was used involved finding the rank of a family of matrices generated by the Adams operations.

Dorette Pronk, Dalhousie University
"Π_2 **and L^H, a tale of two localizations**"
Abstract:I will discuss the relationship between the hammock localization as defined by Dwyer and Kan, and $\Pi_2(\mathcal{C}, W)$, the 2-category obtained by adding right adjoints to the arrows in W.

Oliver Roendigs, University of Western Ontario
"Algebraic K-theory of motivic spaces"
Abstract: Waldhausen's algebraic K-theory of spaces can be applied to the new spaces coming up in motivic homotopy theory. In particular, there is a spectrum $A(k)$ for every field k. If k admits a complex embedding, then $A(k)$ contains Waldhausen's $A(*)$ as a retract. In my talk I will discuss some properties of $A(k)$.

Robert Seely
"Coherence of the Double Involution on $*$-Autonomous Categories"
Abstract: Many formulations of proof nets and sequent calculi for Classical Linear Logic (CLL) take it for granted that a type A is "identical" to its double negation $A^{\perp\perp}$. On the other hand, it has been assumed that $*$-autonomous categories are the appropriate semantic models of (the multiplicative fragment of) CLL. However, in general, in a $*$-autonomous category an object A is only "canonically isomorphic" to its double involution A^{**}. This raises the questions whether $*$-autonomous categories do not, after all, provide an accurate semantic model for these proof nets and whether there could be semantically non-identical proofs (or morphisms), which must be identified in any system which assumes a type is identical to its double negation.

Fortunately, there is no such semantic gap: in this paper we provide a "coherence theorem" for the double involution on $*$-autonomous categories, which tells us that there is no difference between the up-to-identity approach and the up-to-isomorphism approach, as far as this double-negation problem is concerned. This remains true under the presence of exponentials and/or additives. Our proof is fairly short and simple, and we suspect that this is folklore among specialists, though we are not aware of an explicit treatment of this issue in the literature.

This is joint work of J.R.B. Cockett, M. Hasegawa and R.A.G. Seely

Myles Tierney, Rutgers and Université du Québec à Montréal
"Some remarks on bisimplicial sets"
Abstract: Investigating bisimplicial sets for our book (with Andre Joyal) on simplicial homotopy theory, we established an interesting new Quillen homotopy model structure. We also prove, using a new method of proof, a definitive comparison theorem relating the construction B and Wbar.

Ismar Volic, University of Virginia
"Finite type knot invariants and calculus of functors"
Abstract: The theory of finite type invariants has received much attention in recent years due to the celebrated theorem of Kontsevich which equates them with functions on certain types of chords diagrams. The physics-inspired approach to this theory has thus far mostly been based in integration and combinatorics. In this talk, I will present a topological construction which provides a new point of view on finite type knot invariants. Namely, a certain tower of spaces in Goodwillie's calculus of functors turns out to be a classifying object for those invariants. I will first review the most important definitions and results from finite type theory and then discuss its relation to the Goodwillie tower for the space of knots, whose construction will be explained. I will also mention the possible consequences this relationship may have on finite type theory, as well as, time permitting, the Bott-Taubes integrals of configuration spaces which are central to the proof of the main result.

Mark Weber, University of Ottawa
"Generic morphism, parametric representations and weakly cartesian monads"
Abstract: In this talk two notions, generic morphisms and parametric representations, useful for the analysis of functors arising in enumerative combinatorics, higher dimensional category theory, and logic, will be defined and examined. Applications to the Batanin approach to higher category theory, Joyal species and operads will be described.