Math 235
Written Assignment 5

Solutions to the problems below are due at the beginning of class on Friday, May 15th. Please follow the guidelines in the course policy handout when completing this assignment.

1. Let $k \in \mathbb{N}$. Prove that if $6k+1$, $12k+1$, and $18k+1$ are all prime numbers, then $(6k+1)(12k+1)(18k+1)$ is an absolute pseudoprime.

2. Let p be a prime number. Prove that for any integer a, $p | a^p + (p-1)!a$.

3. Prove that if p is prime and $p \equiv 3 \pmod 4$, then $((p-1)/2)!$ is a solution to the congruence $x^2 - 1 \equiv 0 \pmod p$.

4. Prove that if p and $p+2$ are a pair of twin primes, then

$$4((p-1)! + 1) + p \equiv 0 \pmod{p(p+2)}.$$

5. (a) Prove that $\phi(3n) = 3\phi(n)$ iff $3|n$. The symbol ϕ denotes Euler's phi-function.

(b) Prove that if the integer $n > 1$ has r distinct odd prime factors, then $2^r | \phi(n)$.

6. Let m and n be relatively prime positive integers. Prove that

$$m^{\phi(n)} + n^{\phi(m)} \equiv 1 \pmod{mn}.$$