Math 235
Written Assignment 4

Solutions to the problems below are due at the beginning of class on Friday, May 8th. Please follow the guidelines in the course policy handout when completing this assignment.

Part I. Use mathematical induction to prove the following statements. For a review of induction, read pp. 1-5 of your textbook. The problems in Part I do not require any material from the lecture on May 4th.

1. For all $n \geq 1$, $27 | 2^{5n+1} + 5^{n+2}$.

2. For $n \geq 1$, $(-13)^{n+1} \equiv (-13)^n + (-13)^{n-1} \pmod{181}$.

Part II. The problems in this part are based on Fermat's Little Theorem which will be presented in class on May 4th.

3. Prove that if p and q are distinct primes, then $p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$.

4. Prove that if $\gcd(a, 30) = 1$, then $60 | a^4 + 59$.

5. Assume that p and q are distinct odd primes such that $p - 1 | q - 1$. If $\gcd(a, pq) = 1$, prove that $a^{q-1} \equiv 1 \pmod{pq}$.