

**Math 235**  
**Written Assignment 1**

Solutions to the problems below are due at the beginning of class on Wednesday, April 8th. Please follow the guidelines in the course policy handout when completing this assignment.

1. Prove that if  $a$  and  $b$  are integers, with  $b > 0$ , then there exist unique integers  $q$  and  $r$  satisfying  $a = qb + r$ , where  $3b \leq r < 4b$ .

2. For  $n \in \mathbb{N}$ , prove that the integer  $n(7n^2 + 5)$  is of the form  $6k$  for some  $k \in \mathbb{Z}$ .

3. Given integers  $a$  and  $b$  not both 0, prove that if there exist integers  $x$  and  $y$  for which  $ax + by = \gcd(a, b)$ , then  $\gcd(x, y) = 1$ .

4. For any odd integer  $a$ , prove that  $\gcd(3a, 3a + 2) = 1$ .

5. Let  $a, b, c \in \mathbb{Z}$  with  $a \neq 0$ . Prove that if  $\gcd(a, b) = 1$  and  $\gcd(a, c) = 1$ , then  $\gcd(a, bc) = 1$ .