Math 117-01
Practice Problems for Exam 3

1. Determine the work done by the force field \( \mathbf{F}(x, y) = 2x \mathbf{i} \) on an object moving in a straight line from \((-1, 1)\) to \((2, 4)\).

2. Use Green’s theorem to evaluate \( \int_C y^2 \, dx - (e^{4y^2} + 2x) \, dy \) where \( C \) is the rectangle with vertices \((1, 0)\), \((2, 0)\), \((2, 3)\), and \((1, 3)\) oriented counterclockwise.

3. (a) The vector field \( \mathbf{F}(x, y) = (y \cos(xy)) \mathbf{i} + (x \cos(xy) + 6y) \mathbf{j} \) is conservative. Find a potential function for \( \mathbf{F} \).

   (b) Let \( C \) be the portion of the curve \( y = \cos(x) \) from \((\pi, -1)\) to \((2\pi, 1)\). Use your answer from (a) to evaluate the line integral \( \int_C (y \cos(xy)) \, dx + (x \cos(xy) + 6y) \, dy \).

4. Suppose that \( \mathbf{F}(x, y, z) = f(x, y, z) \mathbf{i} + g(x, y, z) \mathbf{j} + h(x, y, z) \mathbf{k} \) is a conservative vector field where \( f \), \( g \), and \( h \) have continuous partial derivatives. Prove that
   \[
   \frac{\partial g}{\partial z} = \frac{\partial h}{\partial y}.
   \]

5. (a) Find parametric equations for the portion of the sphere \( x^2 + y^2 + z^2 = 16 \) that lies above the cone \( z = \sqrt{x^2 + y^2} \). Be sure to include the range of values for the parameters.

   (b) Find parametric equations for the portion of the cylinder \( y^2 + z^2 = 9 \) that lies between the planes \( x = 1 \) and \( x = 4 \). Be sure to include the range of values for the parameters.

6. Evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( \mathbf{F}(x, y) = (e^{-x}) \mathbf{i} + (e^{2y}) \mathbf{j} \) and \( C \) is the path determined by \( \mathbf{r}(t) = 2t \mathbf{i} + 3t \mathbf{j} \), \( 0 \leq t \leq 2 \).

7. Find the area of the region swept out by the line from the origin to the ellipse \( x = a \cos t \), \( y = b \sin t \) if \( t \) varies from \( t = 0 \) to \( t = \pi/4 \). (Here \( a \) and \( b \) are constants with \( a, b > 0 \).)