Math 313/513, Homework 10 (due Thurs Apr. 12th)

Name:__________________________________ 313 or 513 (circle)

Reading

- Read sections 8.5 and 6.4 of Strang.

Problems Math 313:

1. Strang, section 8.5, problem 1
2. Strang, section 8.5, problem 2 (use the L^2 inner product we discussed in class)
3. Find the Fourier series of $f(x) = \cos(3x)$. (Hint: it won't require much computation.)
4. Find the Fourier series of $f(x) = x(2\pi - x)$. You may use an outside tool (e.g., Wolfram Alpha) to evaluate the integrals. Graph this function in MATLAB, along with its Fourier series up through $k = 5$. Include a printout of your results.
5. Find the Fourier series of the function $f(x)$ that equals 0 for $0 \le x \le \pi$ and equals $x - \pi$ for $\pi \le x \le 2\pi$. Also graph this function in MATLAB, along with its Fourier series up through $k = 5$. Include a printout of your results.
6. The spectral theorem says that if A is symmetric, then A is orthogonally diagonalizable. Prove the converse: i.e., that if A is orthogonally diagonalizable, then A must be symmetric.
7. True or false? If true, give a brief explanation. If false, provide a counterexample:
   (a) If A is symmetric, then A has distinct real eigenvalues.
   (b) If A is symmetric, then A is diagonalizable.
   (c) If A is symmetric with eigenvalues 3, 2, 1, then any two eigenvectors that are linearly independent are also orthogonal.
   (d) If A is symmetric with eigenvalues 3, 2, 2, then any two eigenvectors that are linearly independent are also orthogonal.
8. Find a matrix that is orthogonal and symmetric, but not the identity.
9. Strang, section 6.4, problem 26
10. Orthogonally diagonalize the following matrix:

$$\begin{bmatrix} 16 & -4 \\ -4 & 1 \end{bmatrix}$$

(OVER)

11. Orthogonally diagonalize the following matrix. Hint: the eigenvalues are 7 and -2:

$$\begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

(Don't forget to check your answer at the end!)

Math 513: all of the above, plus:

1. Let V be the vector space of $n \times n$ matrices. Prove that $\langle A, B\rangle = \operatorname{tr}(A^T B)$ defines an inner product on V. Also prove that the subspace of symmetric matrices is orthogonal to the subspace of skew-symmetric matrices using this inner product. (Optional challenge: can you write down a formula for the orthogonal projection of a matrix A onto the subspace of symmetric matrices? And projection onto the subspace of skew-symmetric matrices?)

2. Let V be the vector space of continuous functions $f(x)$ on the interval $[0, P]$, for some constant $P > 0$. Define an inner product on V as:

$$\langle f(x), g(x)\rangle = \int_0^P f(x)g(x)dx.$$

Start with the orthogonal functions $1, \sin\left(\frac{2\pi}{P}mx\right), \sin\left(\frac{2\pi}{P}nx\right)$ for $m, n \geq 1$ and make them orthonormal. Find the formula for the Fourier coefficients of a function $f(x)$ defined on $[0, P]$. Hint: your answer should reduce to that given in class when $P = 2\pi$.

MATLAB assignment
To be announced soon!