Math 313/513, Homework 9 (due Thurs. Mar. 29)

Name:__________________________________ 313 or 513 (circle)

Reading

- Read sections 4.1 – 4.4 of Strang.

Problems Math 313:

1. Strang, section 4.1, problem 4
2. Strang, section 4.1, problem 11 (he is referring to the four subspaces)
3. Strang, section 4.1, problem 24
4. Strang, section 4.2, problem 1
5. Strang, section 4.2, problem 8
6. Strang, section 4.2, problem 11
7. Strang, section 4.2, problem 16
8. Strang, section 4.2, problem 17
9. Let A be a matrix with linearly independent columns. Let P be the projection matrix defined in class or on page 210 of Strang.
   (a) Using matrix algebra, check that $P^2 = P$.
   (b) Using geometric reasoning and no algebra, explain why $P^2 = P$.
   (c) Using algebra, check that $P^T = P$.
10. Recall that a matrix A is orthogonal if $A^T A = I$. What are the possible eigenvalues of an $n \times n$ orthogonal matrix? Explain.
11. Suppose A is an $n \times n$ skew-symmetric matrix, meaning that $A^T = -A$. Show that if $\vec{x}$ is any vector in $\mathbb{R}^n$, then $A\vec{x}$ is orthogonal to $\vec{x}$. (We've seen this already when A is the rotation matrix `[0 -1; 1 0]`.) Hint 1: remember $\vec{x} \cdot \vec{y} = (\vec{x})^T \vec{y}$. Hint 2: 0 is the only number that equals its own negative.
12. Find Jim Hefferon's linear algebra textbook online at `http://joshua.smcvt.edu/linearalgebra` and do problem 7 on page 264 of that text. **In this problem and the next, feel free to use MATLAB, but please include a printout or write down by hand any code you use.** However, please don't use MATLAB's built-in code to find lines of best fit.
(OVER)

13. Problem 8 on page 265 of Hefferon.

14. Find **all** least-squares solutions of of $A\vec{x} = \vec{b}$, where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \qquad \vec{b} = \begin{bmatrix} 7 \\ 2 \\ 3 \\ 6 \\ 5 \\ 4 \end{bmatrix}$$

    Feel free to use MATLAB to do matrix computations, but be sure to write down on your assignment what exactly you're doing.

15. Find an orthonormal basis of the column space of the matrix

$$A = \begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & 4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{bmatrix}$$

16. Find a QR factorization of the matrix in the previous problem.

17. Strang, section 4.4, problem 5

18. Strang, section 4.4, problem 7

19. Strang, section 4.4, problem 21

Math 513: all of the above, plus:

1. Suppose A is a matrix with $A^2 = A$. What are the possible eigenvalues of A? Explain. If A has no eigenvalues of zero, prove that A is the identity.

2. Find a 2×2 matrix A such that $A^6 = I$, but $A^k \neq I$ for $k = 1, 2, 3, 4, 5$. Hint: consider rotations in $\mathbb{R}^2$.