Math 313/513, Homework 7 (due Thurs. Mar. 15)

Name:________________________________ 313 or 513 (circle)

Reading

- Read section 6.1 of your text, and take a look at the article *The Linear Algebra Behind Google* by Bryan and Leise (linked on the course web page).

Book problems

- Math 313:
  - Section 6.1, problem 4
  - Section 6.1, problem 19 (explain how you get your answers!)
  - Section 6.1, problem 27
  - Section 6.1, problem 29

1. By hand, find all eigenvalues of, and compute bases for all eigenspaces of the following matrix:
$$A = \begin{bmatrix} -1 & 3 & -4 \\ 2 & -2 & 4 \\ 2 & -3 & 5 \end{bmatrix}$$

2. Suppose A has characteristic polynomial $\lambda(\lambda-1)(\lambda+2)(\lambda-5)$. Explain your answers to the following questions:
   a. What is the determinant of A?
   b. What is the trace of A?
   c. What is the rank of A?
   d. How many linearly independent eigenvectors must A have?

3. Suppose A has characteristic polynomial $-(\lambda-3)^2(\lambda+4)$. Explain your answers to the following questions:
   a. What is the determinant of A?
   b. What is the trace of A?
   c. What is the rank of A?
   d. How many linearly independent eigenvectors *could* A *possibly* have? Why?

4. The predator–prey discrete dynamical system we discussed in class was described by the matrix
$$A = \begin{bmatrix} 0.5 & 0.4 \\ -0.104 & 1.1 \end{bmatrix},$$
where $p = 0.104$ represented the rate at which the rats were consumed by the owls. We found that in the long run, both populations grew exponentially.

a. Suppose that p is replaced with 0.14. Use MATLAB's `eig` command to determine the new eigenvalues and corresponding eigenvectors of A (you may wish to run `help eig`). Write down what these eigenvalues and eigenvectors are.
b. Explain why, in the long run, both populations will decline in this new model.
c. In the long run, what is the ratio of owls to rats?

- Math 513: all of the above, plus:
  - Let λ be an eigenvalue of an $n \times n$ matrix A. Prove that the λ-eigenspace (the set of eigenvectors with eigenvalue λ) is indeed a subspace of $\mathbb{R}^n$.
  - Section 6.1, problem 36

MATLAB assignment

This week, I ask that you implement Google's PageRank algorithm as follows. Write a matlab file (not a function) `lastname_pagerank.m` that loads an $n \times n$ matrix W that is stored in a separate file `W.dat` (there's a link to such a file on course webpage). Use MATLAB's command `load W.dat` to store this matrix in a variable W.

W will be a square $n \times n$ matrix W consisting of 1's and 0's. This is the "web matrix" that has, in the ith row and jth column, a 1 if webpage j links to webpage i. (We'll assume a page will never link to itself.)

First, use W to generate the matrix A we discussed in class (please use *at most* one `for` loop for this). Then generate B by taking 0.85 times A plus 0.15 times the matrix whose entries are all $1/n$. (Recall both A and B should be Markov matrices.)

Find the ranking vector x as the steady-state vector of B (remember, the entries of x should sum to 1). You may reuse the code you wrote in the previous assignment to find steady-state vectors, or you can look into the `eig` command.

Now, x stores the rankings. In your comments, indicate which webpage has the highest rank, and which has the lowest *using the file* `W.dat` on the course web page (which is a 30×30 matrix).

Finally, consider the following scenario. Page 3 is not satisfied with its ranking score, so it creates page 31. Page 3 makes a link to 31, and 31 links back to 3 (and no other changes occur). Write code that makes the necessary changes to W (you should only need two lines for this). In your comments, indicate what page 3's score was before and after this change.

Include your comments, and submit your code to Blackboard. Please remember to name your file in the form `lastname_pagerank.m`