Math 313/513, Homework 4 (due Thurs. Feb. 9)

Name:__________________________________ 313 or 513 (circle)

Reading

- Read sections 3.1 – 3.3.

Book problems

- Math 313:
  - section 3.1: 5, 10, 17, 19, 23, 29 (be sure to justify your answers, especially for 23!)
  - section 3.2: 2, 3, 9, 12, 13, 16
  - section 3.3: 2, 8
  - Recall that $\mathbb{P}_n$ is the vector space of polynomials of degree at most n. Fix a real constant c. Let W_c be the subset of $\mathbb{P}_n$ consisting of polynomials whose constant term equals c. For what values of c is W_c a *subspace* of $\mathbb{P}_n$? Explain.
- Math 513: all of the above, plus:
  - section 3.1: problem 30
  - section 3.2: problem 35 (explain your answer!)
  - Prove carefully that the column space of an $m \times n$ matrix A is indeed a subspace (of $\mathbb{R}^m$). Hint: there are three things you need to check.
  - section

MATLAB assignment

This week we will study $n \times n$ matrices A that are *sparse*, meaning that most entries (say at least 90%) are zero. Such matrices often show up in practice, and MATLAB has a more efficient way of dealing with them: rather than recording n^2 numbers, it keeps track simply of the rows and columns of the nonzero entries, and their values. If A is an ordinary matrix, `sparse(A)` returns a new data type that represents A being stored in this fashion.

For now, we will focus on k-banded square matrices A: this means there exists a number k such that $a_{ij} = 0$ if $|i - j| > k$. Usually, k is much smaller than n, making the matrix sparse.

- In your comments, explain by what familiar term we refer to 0-banded matrices. (OVER)

- Write a function (see hint below) `rand_band(n,k)` that takes in integers $n > 0$ and $k \geq 0$ and outputs a randomly generated $n \times n$ matrix that is k-banded. Be sure that the matrix you return is stored in the `sparse` format.

- Write code, similar to that in last week's assignment, that measures the running time for solving $A\vec{x} = \vec{b}$ for random $n \times 1$ vectors $\vec{b}$ and random 10-banded $n \times n$ matrices A. Let n start at 100 and go up in steps of, say, 50, until it gets so big that your computer and/or MATLAB has difficulty with it. (Warning: don't measure the time it takes to generate the random matrix and vector.)

- Generate a plot with the values of n on the x-axis and the corresponding running times on the y-axis.

- In your comments, make a conjecture based on your results about how the running time depends on n (e.g., is it proportional to n, n^2, n^3, $n \log(n)$, etc.). The point of this exercise is that you can often beat out the "n^3" rule if your matrix is sparse (or banded, in this case).

Now, put all your code together into a `.m` file, being sure to include your comments. Submit to Blackboard, using the format `lastname_hw04.m`

Some MATLAB hints

1. How do you write a function in MATLAB? Ideally, you'd do this in a separate `.m` file, but for now, at the top of your `.m` file, do something like:

   ```
   function z = multiply_numbers(m,n)
         z = m*n;
   end
   ```

   Then put your usual code below. For future reference, you can return multiple values from a function by:

   ```
   function [y, z] = add_and_multiply_numbers(m,n)
         y = m + n;
         z = m*n;
   end
   ```