Math 313/513, Homework 2 (due Thurs. Jan. 26)

Name:_____ 313 or 513 (circle)

Reading

- Skim section 2.1 of Strang, read section 2.2.

Book problems

- Math 313: From **section 2.1: 26, 27, 28. From section 2.2: 10.** Also the problems below:

  1. Find the reduced echelon form of the following matrix:

  $$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix},$$

  and box the pivots.

  2. Find the general solution of the linear system whose augmented matrix is that given in the previous question.
  3. What happens to your answer if the 9 in the matrix is changed to 10?
  4. Solve the linear system whose augmented matrix is given by:

  $$\begin{bmatrix} 1 & 2 & -5 & -6 & 0 & -5 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

  Be sure to state which variables are free, and draw boxes around the pivots.

  5. Find all solutions (if any) of the following linear system:

  $$\begin{aligned} x_1 - 3x_2 + 4x_3 &= -4 \\ 3x_1 - 7x_2 + 7x_3 &= -8 \\ -4x_1 + 6x_2 - x_3 &= 7 \end{aligned}$$

- Math 513: all of the above, plus section 2.1: 32, section 2.2: 19

MATLAB problems

1. Consider the problem of finding the coefficients of the fourth-degree polynomial

$$a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4$$

that passes through five specified points. Assume these data points are given as a 5×2 matrix:

$$\texttt{datapoints} = \begin{bmatrix} 1 & 1 \\ -2 & 3 \\ 0 & 2 \\ 3 & -2 \\ 4 & 9 \end{bmatrix},$$

meaning that the graph of the polynomial passes through the points $(1, 1)$, $(-2, 3)$, etc. Recall from class we can phrase this problem as solving a system of linear equations, say of the form $A\vec{x} = \vec{b}$, where $\vec{x}$ will consist of the (unknown) numbers $a_0, a_1, \ldots, a_4$.

Write a MATLAB `.m` file that does the following, **without using any loops**!

- Define the column vector `b` using `datapoints` (see hint 1 below).
- Define the 5×5 matrix `A`. Remember the first column will consist of all ones (see hint 2 below), the second column will consist of all the t-coordinates of the data points, the third column will consist of all the squares of the t-coordinates of the data points, etc. (see hints 3 and 4).
- In MATLAB, solve the system for `x` using the command `A\b`.
- Generate a plot of the polynomial over the range $-5 \leq t \leq 5$. (Choose a reasonable step size for t.)
- Finally, overlay your plot with a plots of the 5 data points. Make sure your polynomial passes through all the points! (See hints 5 and 6.)
- Be sure to comment your code, so someone else could follow your work.

Some MATLAB hints

1. If you have a matrix M, then $M(:, 2)$ returns the second column of it
2. `ones(m,1)` creates a column vector with m entries of all 1s.
3. To take the third power of each entry of a vector `t`, use `t.^ 3`, not `t^ 3`.
4. If you have some column vectors `u, v, w` of the same length m, you can concatenate them like `[u v w]` to make an $m \times 3$ matrix, for instance.
5. The command `hold on` prevents the current plot from being erased when you plot something new. There's also `hold off`.
6. The function `scatter` takes in two vectors and makes a plot of those points in the plane.