Math 312 (Linear Algebra): Course Summary
Spring 2011, Prof. Jauregui

- Linear systems
  - consistency/inconsistency
  - multiple interpretations of $A\vec{x} = \vec{b}$: matrix equation, vector equation, system of scalar equations
  - row reduction
    - (reduced) echelon form
    - pivot positions, pivot columns
  - vector algebra in $\mathbb{R}^n$
    - linear independence/dependence
    - span of a collection of vectors
  - homogeneous equation
- Linear transformations
  - dynamical interpretation of $A\vec{x} = \vec{b}$
  - the matrix of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$
  - one-to-one and onto linear transformations
  - invertible matrix theorem ("big theorem")
  - interpretation of matrix multiplication in terms of composition
  - geometric interpretation of determinant
- Vector spaces
  - subspaces of $\mathbb{R}^n$
    - column space, null space
  - basis for a subspace of $\mathbb{R}^n$, coordinate vectors
  - rank–nullity theorem
  - abstract vector spaces
    - examples: function spaces, polynomial spaces $\mathbb{P}_n$
    - linear transformations between vector spaces, kernel and range
    - bases for general vector spaces
  - change-of-basis (for vectors and for linear transformations)
- Spectral Theory ("eigenstuff")
  - Eigenvalues and eigenvectors, characteristic polynomial
  - eigenspaces and dimension
  - diagonalization of a square matrix
    - equivalent condition: maximum number of linearly independent eigenvectors
    - sufficient condition: distinct eigenvalues
    - sufficient condition: symmetric matrix

  - interpretation of diagonalization via change-of-basis
  - complex eigenvalues and eigenvectors

- Orthogonality
  - dot product on $\mathbb{R}^n$, orthogonality, orthogonal matrices
  - orthogonal complement of a subspace
  - symmetric matrices and orthogonal diagonalization
  - orthogonal projection
  - orthogonal/orthonormal bases and Gram–Schmidt algorithm
  - least-squares solutions of inconsistent linear systems, error
  - singular values of an $m \times n$ matrix and their interpretation

- Matrix factorizations/decompositions
  - LU decomposition, applications to solving $A\vec{x} = \vec{b}$
  - QR decomposition
  - singular value decomposition (SVD)
  - Jordan canonical form

- Applications
  - construct lines/parabolas/polynomials passing through prescribed points
  - Markov chains, transition of states, steady-state vectors
  - discrete dynamical systems
  - Google's PageRank algorithm
  - construct lines of best fit via least-squares
  - using the SVD to compress images (rank-k approximation to a matrix)
  - principal component analysis
    * applications to finance and data analysis
  - Assignment problem, solution via the Hungarian algorithm
  - matrix exponential, solutions of a systems of linear ODEs
  - error-correcting codes