Union College Math Conference: Applied Topology

September 13–15, 2019

Friday program

4:30–5:30pm: Reception (Olin Rotunda)

5:30–6:30pm: Plenary talk: John McCleary, *The Oberwolfach archives and topology* (Olin 115)

Saturday program

8:00–9:00am: Coffee & pastries, registration (Olin Rotunda)

Session I (Lippman 014)
- 9:00–9:25 Sara Kalisnik: *Strengthening topological signal with lenses*
- 9:30–9:55 Vladimir Itskov: *CANCELED*
- 10:00–10:25 Donald Sheehy: *The cohomology of impossible figures, revisited*

10:30–11:00am: Coffee break (Olin Rotunda)

11:00am–12:00pm: Plenary talk: Álvaro Lozano-Robledo, *Recent progress on the classification of torsion subgroups of elliptic curves* (Olin 115)

12:00–2:00pm: Lunch break

Session II (Lippman 014)
- 2:00–2:25 Matthew Zaremsky: *Bestvina–Brady Morse theory on Vietoris–Rips complexes*
- 2:30–2:55 Nicholas Scoville: *On the automorphism group of the Morse complex*
- 3:00–3:25 Érika Roldán Roa: *Two geometric problems in extremal topological combinatorics*
- 3:30–3:55 Jakob Hansen: *Laplacians of cellular sheaves and their applications*

4:00–4:30pm: Coffee break (Olin Rotunda)

4:30–5:30pm: Plenary talk: Robert Ghrist, *Applied Topology: Sheaves and Cosheaves* (Olin 115)

6:00–8:30pm: Conference dinner (Old Chapel)

Sunday program

8:00–8:30am: Coffee & pastries (Olin Rotunda)

Session III (Lippman 014)

8:30–8:55 Chad Giusti: *Path signatures and neural data analysis*

9:00–9:25 Sarah Tymochko: *Adaptive partitioning of persistence diagrams for featurization using template functions*

9:30–9:55 Shelley Kandola: *The topological complexity of spaces of digital images*

10:00–10:25 Christopher Tralie: *Designer Takens: A tale of twisted time series*

10:30–11:00am: Coffee break (Olin Rotunda)

11:00am–12:00pm: Plenary talk: Carolyn Gordon, *Decoding geometry and topology from the Steklov spectrum of orbisurfaces* (Olin 115)

12:00–1:45pm: Lunch break

Session IV (Lippman 014)

1:45–2:10 Luis Scoccola: *Homotopy interleavings*

2:15–2:40 Woojin Kim: *Generalized persistence diagrams for persistence modules over posets*

2:45–3:10 Ashleigh Thomas: *Modifying multipersistence distances to differentiate between mortal and immortal persistence features*

3:15–3:40 Alex McCleary: *Multiparameter persistent homology*

Abstracts

Robert Ghrist
Plenary talk: *Applied Topology: Sheaves & Cosheaves*

The recent success of persistent homology in describing large scale features of point cloud data is just the beginning of what algebraic topology is good for. This talk will survey work in applications of sheaves and cosheaves — algebraic data structures which are tethered to a space — in networks, sensing, optimization, and more.

Sara Kalisnik
Strengthening topological signal with lenses

A common problem in data science is to determine properties of a space from a sample. For instance, under certain assumptions a subspace of a Euclidean space may be homotopy equivalent to the union of balls around sample points, which is in turn homotopy equivalent to the Čech complex of the sample. This enables us to determine the unknown space up to homotopy type, in particular giving us the homology of the space. A seminal result by Niyogi, Smale and Weinberger states that if a sample of a closed smooth submanifold of a Euclidean space is dense enough (relative to the reach of the manifold), there exists an interval of radii for which the union of closed balls around sample points deformation retracts to the manifold. A tangent space is a good local approximation of a manifold, so we can expect that an object, elongated in the tangent direction, will better approximate the manifold than a ball. We present the result that the union of lenses around sample points deformation retracts to the manifold while requiring much smaller density than in the case of union of balls. The proof requires new techniques, as unlike in the case of balls, the normal projection of a union of lenses is in general not a deformation retraction.

Donald Sheehy
The cohomology of impossible figures, revisited

Roger Penrose is credited with identifying nontrivial cohomology as a basic requirement for a certain class of optical illusion present in the work of artist M.C. Escher. The so-called Penrose Triangle has since become an emblem for cohomology. It captures, in a discrete way, the kind of rotating vector fields whose singularities allow for the nonexistence of an antiderivative and stand as a primary motivating example for de Rham cohomogy in the first place. However, in many ways, the discrete theory is much older. In this talk, I will explore some appearances of discrete de Rham cohomology theory in the 1860's, the 1960's, and their impact in later problems in discrete and computational geometry. Then, I will argue that discrete differential forms are a powerful tool for geometric graph algorithms.

Matthew Zaremsky
Bestvina–Brady Morse theory on Vietoris–Rips complexes

Bestvina–Brady discrete Morse theory is a powerful tool for leveraging “local topological information about a cell complex to make “global topological conclusions. Over the last two decades it has proven invaluable in geometric group theory, but only in the last year or so has it been brought to the attention of topological data analysts, as a powerful generalization of

Forman's more well known discrete Morse theory. This talk will serve as an "invitation" to Bestvina–Brady discrete Morse theory, with a focus on applications to Vietoris–Rips complexes of metric spaces.

Nicholas Scoville
On the automorphism group of the Morse complex

Let K be a finite, connected, abstract simplicial complex. The Morse complex $\mathcal{M}(K)$, first introduced by Chari and Joswig, is the simplicial complex constructed from all gradient vector fields on K. In this talk, we discuss recent progress computing the automorphism group of $\mathcal{M}(K)$. We show that if K is neither the boundary of the n-simplex nor a cycle, then $\mathrm{Aut}(\mathcal{M}(K)) \cong \mathrm{Aut}(K)$. In the case where $K = C_n$, a cycle of length n, we show that $\mathrm{Aut}(\mathcal{M}(C_n)) \cong \mathrm{Aut}(C_{2n})$. When $K = \partial\Delta^n$, we prove that $\mathrm{Aut}(\mathcal{M}(\partial^n)) \cong \mathrm{Aut}(\partial^n) \times \mathbb{Z}_2$. These results are based on recent work of Capitelli and Minian.

Érika Roldán Roa
Two geometric problems in extremal topological combinatorics

We consider two optimization questions with respect to regular polyforms (polyiamonds and polyominoes). What is the maximum number of holes that a polyform with n tiles can enclose, and what is the minimum number of tiles required to construct a polyform with h holes? We completely solved this problem for polyiamonds with the construction of a sequence of polyiamonds that reaches the maximum number of holes for any given number of tiles. In the case of polyominoes we give the first and second order asymptotics of the maximum number of holes that a polyomino with n tiles can have and we exhibit a sequence of polyominoes that reaches this topological maximum. These results give an upper bound for the expectation of the rank of the first homology group of random polyiamonds and random polyominoes (a lower bound is also known and it is also linear with respect to the number of tiles). Joint work with Matthew Kahle (polyominoes) and Greg Malen (polyiamonds).

Jakob Hansen
Laplacians of cellular sheaves and their applications

A sheaf of inner product spaces on a regular cell complex has a set of canonically associated Hodge Laplacians computing the cohomology of the sheaf. These linear operators generalize the Laplacians of graphs and simplicial complexes, and offer a way to connect discrete algebraic topological aspects of cellular sheaves with more continuous, geometric concepts. The use of the graph Laplacian in systems engineering and data analysis is an example of this relationship for the constant sheaf. Other special cases of sheaf Laplacians (e.g., graph connection Laplacians and Laplacians of matrix-weighted graphs) have also been used to attack problems in both fields. Interpreting problems associated with networks and data in terms of associated cellular sheaves deepens understanding and inspires extensions of these methods to new problems.

In this talk, I will introduce cellular sheaves together with their Laplacians and spectral theory, and discuss concrete applications of this theory to two problems: learning a network structure from data and distributed convex optimization.

Chad Giusti

Path signatures and neural data analysis

Path signatures provide an embedding of the space of paths in Euclidean space in the tensor algebra on that space, and satisfy a number of properties that make them powerful tools for data analysis. We briefly sketch this tool kit and potential uses, then give an example of its use in the study of context of human EEG recordings.

Sarah Tymochko

Adaptive partitioning of persistence diagrams for featurization using template functions

Tools from the field of Topological Data Analysis, specifically persistent homology, have found continued success in applications, thus there has been increasing interest in developing methods to use persistence diagrams as inputs for machine learning techniques. Doing so requires some mathematical creativity, as the space of persistence diagrams does not have the desired properties for machine learning. One such method of featurizing persistence diagrams developed by Perea, Munch, and Khasawneh uses continuous and compactly supported functions, referred to as“template functions’ to vectorize information from the persistence diagrams. In this talk, we will provide a method of adaptively partitioning persistence diagrams to create featurizations from template functions based on localized information. Additionally we will show results in application to some example datasets.

Shelley Kandola

The topological complexity of spaces of digital images

In this talk, I present my recently-completed dissertation on the topological complexity of spaces of digital images. The images I focus on are those that have been segmented by digital Jordan curves as a means of image compression. In particular, I focus on digital images that are represented by Jordan curves with the Khalimsky topology. Using that underlying topology, I interpret the set of digital Jordan curves as a finite topological space. In fact, this space is path connected. From here I explore two applications: The primary application is topological complexity. Although it has its roots in the robot motion planning problem, computing the topological complexity of this space determines the minimal number of motion planning rules required to continuously morph one image into another, and determining the associated motion planners provides the specific image processing algorithms for doing so. Lastly, I discuss the role that this could play in image and character recognition.

Christopher Tralie

Designer Takens: A tale of twisted time series

Takens’ theorem provides conditions under which one can reconstruct an attractor of a dynamical system up to a homeomophism from a single observable. This has been used in conjunction with persistent homology to show, for instance, that periodic time series give rise to loops and quasiperiodic time series give rise to torii. In this work, we demonstrate a strategy to reverse engineer Takens’ theorem to obtain time series associated not only to these and other orientable manifolds such as there sphere and 2-holed torus, but also to the Klein bottle and $\mathbb{RP}^2$. In addition to persistent homology, we use Perea’s projective coordinates to verify that we have indeed recovered these twisted spaces from the time series data. We also use this as an opportunity to showcase an upcoming Python library by Tralie and Perea, "Dimension Reduction with Eilenberg-MacLane Coordinates" (DREiMac), which implements circular and projective coordinates.

Luis Scoccola
Homotopy interleavings

We combine Blumberg–Lesnick's homotopy interleaving distance with the theory of interleavings in categories with a flow of de Silva, Munch, and Stefanou, to get a theory of homotopy interleavings in $PSh(I)-$enriched model categories, for I a suitable monoidal poset.

In this framework, the Gromov–Hausdorff distance is a homotopy interleaving distance, and the continuity of the Vietoris–Rips functor follows from a few abstract arguments.

Woojin Kim
Generalized persistence diagrams for persistence modules over posets

When a category C satisfies certain conditions, we define the notion of rank invariant for arbitrary poset-indexed functors $F : P \to C$ from a category theory perspective. This generalizes the standard notion of rank invariant as well as Patel's recent extension. Specifically, the barcode of any interval decomposable persistence modules $F : P \to vec$ of finite dimensional vector spaces can be extracted from the rank invariant by the principle of inclusion-exclusion. Generalizing this idea allows freedom of choosing the indexing poset P of $F : P \to C$ in defining Patel's generalized persistence diagram of F. By specializing our idea to zigzag persistence modules, we also show that the zigzag barcode of a Reeb graph can be obtained in a purely set-theoretic setting without passing to the category of vector spaces. This leads to a promotion of Patel's semicontinuity theorem about type-A persistence diagrams to a Lipschitz continuity theorem for the category of sets.

Ashleigh Thomas
Modifying multipersistence distances to differentiate between mortal and immortal persistence features

This talk will give a geometric perspective on a primary-decomposition-based modification to existing multipersistence distances. The resulting modified distances treat qualitatively different persistence features (homology classes that are mortal vs those that are immortal) separately so that persistence modules with unbounded support can be within finite distance of other persistence modules. Modules with unbounded support include (but are not limited to) zeroth persistence modules and modules with features that do not depend on every persistence parameter.

This primary distance modification works for distances based on the Hilbert and rank functions as well as the interleaving distance. Explicit descriptions the modified Hilbert, rank, and interleaving distances will be presented. This is joint work with Ezra Miller.

Alex McCleary
Multiparameter persistent homology

Typically persistent homology starts with a filtered simplicial complex and returns a discrete invariant, the persistence diagram, which characterizes where holes are born and die in the filtration. In this talk we will extend this concept to the setting of multifiltrations, where the simplicial complex is filtered by more than one parameter.