

Final Exam Practice — November 2009
Math 113

- You may use one 8.5×11 sheet of paper with notes (both sides, if you wish). Calculators or computers are not allowed.
 - The final exam will be Thursday, November 19, 11:30am–1:30pm, in HUMN 019.
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(1) Find the derivatives of the following functions.

(a) $f(x) = \frac{e^x - e^{-x}}{2}$

(b) $h(u) = \arcsin(\sqrt{u})$

(2) Integrate:

(a) $\int (x\sqrt{x} + 3e^{-x}) dx$

(b) $\int_1^x \frac{1}{t} dt$ ($x > 0$)

(c) $\int_0^\infty \frac{x}{(1+x^2)^2} dx$

(d) $\int e^x \sin x dx$

(e) $\int \frac{\sin(\ln t)}{t} dt$

(f) $\int_0^1 \sin(\ln t) dt$ (Quite hard.)

(3) Consider the curve $(x-2)^2 + y^2 = 1$ (a circle of radius 1 centered at (2,1)).

(a) Find $\frac{dy}{dx}$ for this curve.

(b) Suppose you draw a line through the origin, with positive slope, that is tangent to the circle. What is the equation for the line? (Hint: The line through the origin and the point (x,y) has slope y/x , and this must equal $\frac{dy}{dx}$ for the curve for the line to be tangent.)

(4) You are flying a kite at a (constant) height of 300ft. How fast is the kite travelling downwind if there is currently 500ft of string out and you are letting string out at 2 ft/s? (Pretend that there is no sagging in the string.)

(5) You are designing the layout for a book. Each right-facing page will have margins of 1 inch on the top, bottom, and right, and 2 inches on the left. (The left-facing pages are a mirror image of this.) If you wish to have 24 square inches of printable area on each page, what dimensions should be used to minimize the amount of paper used?

(6) Evaluate the following limits.

(a) $\lim_{x \rightarrow 0} \frac{x(1-\cos x)}{x-\sin x}$

(b) $\lim_{\theta \rightarrow \pi/2^-} (\sec^3 \theta - \tan^3 \theta)$

(7) (a) Estimate $\frac{1}{1.04}$ using the linearization of $f(x) = 1/x$ at $x = 1$.

(b) Give a bound on the error of your estimate. (Recall that the linearization is a Taylor polynomial of order 1.)

(This semester, we looked at how Newton's method can be used to implement division efficiently on a computer. Another technique for computing $\frac{p}{q}$ efficiently involves approximating reciprocals by using the linearization of $1/x$ at $x = 1$.)

(8) Suppose $f''(x) = e^{-x^2}$. Why can f have no relative maxima?

(9) Consider the region bounded by $y = e^{-x}$, $y = 0$, $x = 0$, and $x = 3$.

(a) Find the volume of the solid obtained by rotating this region about the x -axis.

(b) Find the volume of the solid obtained by rotating this region about the y -axis.

(10) Find the inverse of the function $f(x) = \ln(1+2x)$. What are the domain and range of f ? What are the domain and range of f^{-1} ?

(11) You decide to use Newton's method to find a solution to $2x - x^2 = 0$, and you try $a = 1$ as your initial guess. Why will this not work?

(12) Find the area of the surface generated by rotating the curve $y = x^3$, $0 \leq x \leq 1$, about the x -axis.