Related rates

1. You are on a Ferris wheel with a radius of 50m. How fast is it rotating if your height above the center of the Ferris wheel is currently 25m and increasing at 3m/s?

Taylor and Maclaurin polynomials

2. (a) Find the 2nd-order Maclaurin polynomial for $f(x) = \cos x$ at $x = 0$.
   (b) Use the polynomial to estimate $\cos(0.2)$.
   (c) Use the remainder estimation theorem to give an upper bound on your error.
   (d) Use a computer/calculator to find $\cos(0.2)$. How accurate was the estimate?

3. Consider the initial value problem [1]

$$\begin{aligned} y' &= \tfrac{1}{2}(1-y)y \\ y(0) &= 1/5. \end{aligned}$$

   (a) Find $y'(0)$.
   (b) Find $y''(0)$. (Hint: Since the power rule is cleaner than the product rule, multiply out before taking the derivative.)
   (c) Will the graph of y initially be increasing or decreasing? Concave up or down?
   (d) What is the 2nd-order Taylor polynomial for $y(t)$ centered at $t = 0$ (i.e., what is the 2nd-order Maclaurin polynomial)?
   (e) Use your Taylor polynomial to estimate $y(0.5)$.
   (f) An exact solution to this initial value problem is $y(t) = \dfrac{1}{1+4e^{-t/2}}$. Compare your estimate with the exact value of $y(0.5)$.

[1] This is a simple model for the spread of an infection through a population, where $y(t)$ represents the proportion of the population that is infected at time t. This model assumes that the disease causes no mortality, and that once infected, individuals remain infected—a bit like herpes. Note that y' would represent the rate of new infections, while $1-y$ would represent the proportion of the population that is *not* infected. The term $(1-y)y$ will be proportional to the frequency with which an uninfected person comes in contact with an infected person, which in turn is proportional to the rate of new infections; this is why it takes the form $y' = k(1-y)y$.

Analysis of functions (§4.1, 4.2)

1. Let $f(x) = \sin|x|$, $-2\pi \leq x \leq 2\pi$.
   (a) Sketch the graph of $y = f(x)$.
   (b) What are the critical points of f? Which of these are stationary points?
   (c) Use the 2nd derivative test, where possible, to identify each critical point as a local max, min, or neither. At which critical point does the 2nd derivative test not work?
   (d) At the critical point where the 2nd derivative test failed, use the first derivative test to identify the point as a local max, min, or neither.
2. 4.2 #19