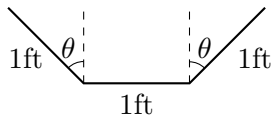


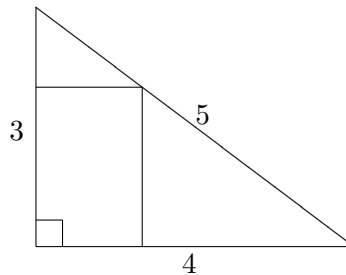
Optimization

1. A farmer is going to make a drinking trough for his cattle with the following cross-section:



What should angle θ be to maximize the volume of the trough? (The length of the trough will be 15ft, but this is irrelevant.)

2. What rectangle drawn inside a 3-4-5 right triangle has maximum area?



Newton's method

3. This exercise looks at the use of Newton's method to find square roots. Fix a number k and let $f(x) = x^2 - k$.
- What are the solutions to $f(x) = 0$?
 - Simplify $N(x) = x - \frac{f(x)}{f'(x)}$.
 - Estimate $\sqrt{2}$ by using $k = 2$ and doing two iterations of Newton's method with an initial guess of $x_0 = 1$ (to find a root of $f(x)$). There is evidence that this method of estimating square roots was known to the Babylonians, though it is unlikely that they referred to it as Newton's method.
 - What happens if you start with $x_0 = -1$?
 - What happens if you try to use $k = -2$?

Hint: You can do many iterations of Newton's method in a spreadsheet without much effort. In cell A1, put your initial guess x_0 . In cell A2, put the formula used for Newton's method in terms of A1; for example, if you were going to use Newton's method to find a root for $f(x) = \sin x$, you would have $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\sin x_n}{\cos x_n} = x_n - \tan(x_n)$, so you would enter “=A1 - TAN(A1)” into cell A2. Then, “fill” the formula downwards: if you select cell A2, a small black square should appear in the bottom right of the cell; grab it and drag it down several cells. This will copy your formula from A2 into those cells, but with the references to A1 adjusted appropriately (so A3 will get “=A2 - TAN(A2)”, A4 will get “=A3 - TAN(A3)”, and so on). If you change your starting value, the other cells will automatically be recalculated; however, if you change the *formula* in A2, the formulas in the cells below will not automatically be updated, so you will have to re-fill. (If the spreadsheet does not display enough digits, you can format the cells to show more digits; if you see ### you probably need to make the column wider.)