Improper integrals, areas, and volumes

1. (a) For $p \leq 1$, show that $\int_1^\infty x^{-p}\,dx = \infty$.
   (b) For $p > 1$, find $\int_1^\infty x^{-p}\,dx$.

2. Find the area of the region bounded by $x - y = 2$ and $x = y^2$.

3. Observe that, for fixed $r > 0$, the curve $y = \sqrt{r^2 - x^2}$ is a semicircle of radius r centered at the origin (since it satisfies $x^2 + y^2 = r^2$, the equation for a circle of radius r), so rotating the region bounded by $y = \sqrt{r^2 - x^2}$ and $y = 0$ about the x-axis sweeps out a sphere of radius r. Find the volume of this sphere.

4. Find the volume obtained by rotating the region bounded by $y = e^{-x^2}$ and $y = 0$ about the y-axis. (Note: You need to be careful not to "double count" the volume, since portions of the region for positive and negative x sweep out the same points in space.)