

Limits more rigorously and continuity (§§1.4–1.5)

1. Let

$$f(x) = \begin{cases} |x| & \text{if } x \text{ is rational,} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

- (a) Sketch the graph of $f(x)$.
 - (b) Prove that f is continuous at 0.
2. Find an interval $[a, b]$ that contains a root of $f(x) = x + e^x$, and explain how you know your interval *must* contain a root.
3. In class we guessed that $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$ based on numerical evidence. But that is not enough to be certain that $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$. Given that $1 - x^2/2 \leq \cos x \leq 1$ for all x , show that $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$. (Hint: No ε - δ proof is required here. Use the squeeze lemma.)
4. Mimic the argument used in class to show $\lim_{x \rightarrow 2} (2x + 1) = 5$ to give a proof that $\lim_{x \rightarrow 1} (3x - 7) = -4$.