1. Suppose \( a > 0 \). Let \( f(x) = e^x \) and \( g(x) = a^x \).

   a. Show that there is a constant \( b \) where \( g(x) = f(bx) \).
   
   (This says that every exponential function can be viewed as coming from \( e^x \) in a natural way.)

   b. On the graph on the next page, carefully draw the graphs of \( y = 4^x \), \( y = 2^x \), \( y = 1^x \), \( y = \left(\frac{1}{2}\right)^x \) and \( y = \left(\frac{1}{4}\right)^x \), and label them.

   c. What values of \( b \) correspond to each of the graphs you drew above?

   d. From your data above, what do you think the graph of \( y = \frac{1}{e^x} \) would look like?
   
   (Hint: What value of \( b \) does this correspond to?)

   e. Make a conjecture about the shape of the graph of \( y = e^{bx} \) for each of the following ranges of \( b \):

      1. \( b > 1 \)
      2. \( 0 < b < 1 \)
      3. \( -1 < b < 0 \)
      4. \( b < -1 \)

   Explain your reasoning.

2. Suppose \( a > 0 \), and let \( x = a^b \) and \( y = a^c \). This says that \( \log_a(x) = b \) and \( \log_a(y) = c \).

   One of the properties of exponents is \((a^b)(a^c) = a^{b+c}\). What does this tell you about the relationship between \( \log_a(x) \), \( \log_a(y) \) and \( \log_a(xy) \)? Explain your reasoning. (Note that we have not yet developed any rules for the logarithm, so you don’t have any to call on. The information given here is all you need.)

   (Hint: What is another way to write \( xy \)? What does the rule of exponents say about this? Finally, recall that \( \log_a(z) \) and \( a^z \) are inverses.)