Math 15H Quiz 3 Comments Fall 2002

In many ways, you did quite well with this. For example, most of you were able to determine the type of surface in problem 1 and to make a reasonable sketch it in class. Although problem 2 was harder, I was pleased to see that many were able to come up with a value of a in part (b). The main problems were in the handling of the details, and in the explanations. Some of you still seem to think that giving an answer with no analysis or explanation is sufficient. It isn't, and I will be taking off more for it in the future.

In question 1, for part (a), I had intended that you would use the signs of the coefficients and the constant to determine the shape of the surface without doing computations (that's why there wasn't a lot of space for it). Most of you did this, but some used traces, which is fine, but more work than needed for this part.

Part (b), was where the details of the traces should be used. For the in class part, it was fine to give a rough sketch based on these traces, but for the take-home part, I expected you to be more accurate both in your traces and in your final sketch. Many of you didn't improve your sketches for the take-home part, and so the relative sizes of various part of your diagram were not accurate. Careful placement of the planes in which the traces occur is crucial, as is getting the shapes of the traces more accurate. You should at least get the endpoints of the hyperbolas in the right place on the graph.

One thing that was confusing to me was that some students computed traces that didn't appear in their sketches. If you go to the trouble of determining the trace, you should show it in the sketch (otherwise, why compute it). This was particularly common for the traces at $z = 1$ and $z = -1$. These are formed by straight lines that cross (like the critical level of the saddle), but few actually drew these in their diagrams. Also, for z values larger than 1 (or smaller than -1), the type of the hyperbola switches, and although some did compute the trace at $z = 2$, few included these traces in their final images. I have included one that did in the notebook outside my office, and several others as well, to give you a variety of examples to work from. There were some nice drawings.

For question 2, in part (a), some didn't draw the intersection, or didn't draw it in with the two surfaces. It really doesn't do much good to draw an ellipse and call that the intersection when the surfaces are drawn somewhere else. Part (b) tells you that it *is* an ellipse, so drawing an ellipse by itself for this part doesn't tell you anything new. It should be drawn in place to help you see how the two surfaces intersect. (Again, I included several of the better drawings in the notebook outside my office.)

When you do these drawings, it is particularly helpful to indicate the over- and under-crossings by gaps in the line that is farther away, as we discussed in class. Without these depth cues, it is very difficult to "read" your image properly, and I had real trouble with several of them. Because you will need to erase in order to do this, it is best to use pencil for these sketches (some are still trying to do this in ink).

In part (b), about half the case were able to determine that $a = \sqrt{3}$ works. (Note that $a = -\sqrt{3}$ also works.) Don't forget, however, to explain how you obtained this. In particular, why the radius of the circle will be 2, and how you know that the center will be at the origin. These are crucial details, and your justification is incomplete without them. There were some nice descriptions of how a affects the position of the plane, which also is important to include. Several students did not find a specific value of a that worked, but were able to make intelligent observations about why such a value should (or should not) exist, and even about its relative size, and they got points for doing so. Those who made erroneous computations, unsupported by explanatory comments, got considerably less.

For part (c), your argument should begin with an explanation of what you think the level set for $g\colon \mathbf{R}^3 \to \mathbf{R}^2$ would *mean*, since we haven't analyzed the level sets for functions of this type, yet. So you should say that the level set at "height" $(4, 0)$ should be the set of points (x, y, z) in $\mathbf{R}^3$ such that $g(x, y, z) = (4, 0)$. Since $g(x, y, z) = (4x^2 + y^2, x - z)$, this would require that $4x^2 + y^2 = 4$ and $x - z = 0$ (simultaneously). Separately, these are the conditions that produce the surfaces in part (a), so the points where both are true are the points on the intersection of these surfaces, namely the ellipse from part (a).

Note that this analysis describes the connection between g itself and the functions f_1 and f_2 from part (a), and points out that the *intersection* is the thing that is required. Several of you claimed that the level set should be the union of the two surfaces, and many were not really clear about *what* part of the sketch for part (a) they thought was related to the level set.

For part (d), you were asked to generalize what you see in this specific case to what happens in general, and (of course), to give some justification for your claim. What we observed in part (c) is that the level set for g at (a, b) is the intersection of the level sets for the coordinate functions of g. That is, the level set for g is the intersection of the level surface for g_1 at height a and g_2 at height b. So the level set is formed by the intersection of two surfaces. Thus, in general, the level sets for $g\colon \mathbf{R}^3 \to \mathbf{R}^2$ will be curves in space.