

Putnam Problems for November 4

The following idea can be applied to solve each of the problems listed below.
How? Hmmmm...

Pigeonhole Principle: If $kn+1$ objects ($k \geq 1$) are distributed among n boxes, one of the boxes will contain at least $k+1$ objects.

1. Given a set of $n+1$ positive integers, none of which exceeds $2n$, show that at least one member of the set must divide another member of the set.
2. Prove that there exist integers a, b, c not all zero and each of absolute value less than one million, such that

$$|a + b\sqrt{2} + c\sqrt{3}| < 10^{-11}.$$

3. Given any set of ten natural numbers between 1 and 99 inclusive, prove that there are two disjoint nonempty subsets of the set with equal sums of their elements.