Putnam Practice Problems for 10/14/03

The first four of the following can be proven using a technique called mathematical induction, but they can also be done using other methods. Even if you know about induction, try another approach. At the beginning of the hour, we'll discuss your ideas and then I'll give you a brief overview of induction. It can be helpful on the Putnam Exam.

1. Prove that $1 + 2 + 3 + 4 + \cdots + n = \dfrac{n(n+1)}{2}$ **for all positive integers** n**.**

2. Develop a similar formula for the sum of the first n **odd numbers. Prove that your formula is true for all positive integers.**

3. Prove that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \cdots + \frac{1}{(n-1) \cdot n} = \frac{n-1}{n}$$

for all integers $n \geq 2$**.**

4. Prove that for all positive integers n**,**

$$1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = \frac{2n^3 + 3n^2 + n}{6}$$

The following question is on last year's Putnam exam

5. Let k **be a fixed positive integer. The** nth **derivative of** $\dfrac{1}{x^k - 1}$ **has the form** $\dfrac{P_n(x)}{(x^k - 1)^{n+1}}$ **where** $P_n(x)$ **is a polynomial. Find** $P_n(1)$**.**