1. The electric field in a region is given by \( \mathbf{E}(x,y,z) = 4\mathbf{i} + 3y^2\mathbf{j} + 5z\mathbf{k} \), where it is measured in N/C and \( x, y, z \) are coordinates. The coordinates are measured in meters. Consider a cubic Gaussian surface bounded by \( x = 1, x = 3, y = 0, y = 2, z = 0, \) and \( z = 2 \).

a. Compute the total electrical flux through the Gaussian Surface.

\[
\Phi = \oint_S \mathbf{E} \cdot d\mathbf{A} = \int_0^2 \int_0^2 \int_1^3 (6y^2 + 5) \, dx \, dy \, dz = 207 \quad \text{(in SI units)}
\]

b. Find the net electrical charge enclosed by the Gaussian surface.

According to Gauss' law:

\[
\Phi = \frac{Q_{net}}{\varepsilon_0}
\]

\[
Q_{net} = \Phi (\varepsilon_0) = \left( 88 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \left( 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \right)
\]

\[
= 7.8 \times 10^{-10} \text{ C}
\]
2. Consider the vector field \( \mathbf{F}(x,y,z) = (x^2y+7x)i + (x^3 \sin z - 2y)j + (4-2xyz)k \). Use the Divergence Theorem to find the flux across the surface of the solid enclosed by the cylinder \( y^2 + z^2 = 16 \) and the planes \( x=2 \) and \( x=5 \), with outward orientation.

\[
\nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}(x^2y+7x) + \frac{\partial}{\partial y}(x^3 \sin z - 2y) + \frac{\partial}{\partial z}(4-2xyz) = 2xy + 7 - 2 - 2xy = 5
\]

\[
\Phi = \iiint_G (\mathbf{F} \cdot \mathbf{n}) \, d\mathbf{S} = \iiint_G (\nabla \cdot \mathbf{F}) \, d\mathbf{V}
\]

\[
= \iiint_G 5 \, d\mathbf{V} = 5 \iiint_G d\mathbf{V}
\]

\[
= 5 \text{ (Volume of G)}.
\]

\( G \) is a cylinder with radius 4 and height 3. So, Volume \( G = (\pi 4^2)3 = 48\pi \)

\[
\Phi = 5 \text{ (Volume of G)}
\]

\[
= (5)(48\pi) = 240\pi
\]
Direction: Show all your work!

1. The electric Field in a region is given by \( \vec{E}(x, y, z) = 4\hat{i} + 3y^2\hat{j} + 5z\hat{k} \), where it is measured in N/C and \( x, y, z \) are coordinates. The coordinates are measured in meters. Consider a cubic Gaussian surface bounded by \( x = 1, x = 3, y = 0, y = 2, z = 0, \) and \( z = 2 \). 

   a. Compute the total electrical flux though the Gaussian Surface.

   \[
   \Phi = \oint \vec{E} \cdot d\vec{A} = \int_{x=1}^{x=3} \int_{y=0}^{y=2} \int_{z=0}^{z=2} E_x \, dx \, dy \, dz \\
   \begin{align*}
   &= \int_{x=1}^{x=3} \int_{y=0}^{y=2} \int_{z=0}^{z=2} (4 + 3y^2 + 5z) \, dx \, dy \, dz \\
   &= \int_{x=1}^{x=3} \int_{y=0}^{y=2} \int_{z=0}^{z=2} 4 \, dx \, dy \, dz + \int_{x=1}^{x=3} \int_{y=0}^{y=2} \int_{z=0}^{z=2} 3y^2 \, dx \, dy \, dz + \int_{x=1}^{x=3} \int_{y=0}^{y=2} \int_{z=0}^{z=2} 5z \, dx \, dy \, dz \\
   &= -16 \, \text{C} \cdot \text{m}^2 + 48 \, \text{C} \cdot \text{m}^2 + 40 \, \text{C} \cdot \text{m}^2 \\
   &= 84 \, \text{C} \cdot \text{m}^2.
   \end{align*}
   \]

   b. Find the net electrical charge enclosed by the Gaussian surface.