Direction: Show all your work!

1. An alpha particle (the nucleus of a helium atom) is placed in a region where the electrical potential is given by $V(x, y, z) = \frac{1 (\text{Volt} \cdot m)}{\sqrt{x^2 + y^2 + z^2}}$, where $x, y,$ and $z$ are the coordinates and are measured in meters. (Note that the mass of an alpha particle is $m = 6.64 \times 10^{-27}$ kg, and it has a charge $q = 3.2 \times 10^{-19}$ C.)

a. What is the electrical potential at $(1m, 2m, 3m)$? (in other words, find $V(1, 2, 3) =$ ?)

$$V(1, 2, 3) = \frac{1 \text{Volt} \cdot m}{\sqrt{(1m)^2 + (2m)^2 + (3m)^2}} = \frac{1 \text{Volt} \cdot m}{\sqrt{14m^2}} = \frac{1}{\sqrt{14}} V$$

b. What would be the electrical potential energy of the alpha particle if it were to be located at $(1m, 2m, 3m)$?

$$U = qV = (3.2 \times 10^{-19} C) \left( \frac{1}{\sqrt{14}} V \right) = \frac{3.2}{\sqrt{14}} \times 10^{-19} J$$

$$U = \frac{2}{\sqrt{14}} \text{ eV}$$

c. Find the electric field in the region at any given point in terms of $x, y,$ and $z$.

$$E = -\nabla V$$

$$E = -\frac{2}{2x} \left( \frac{1}{\sqrt{x^2+y^2+z^2}} \right) \hat{i} -\frac{2}{2y} \left( \frac{1}{\sqrt{x^2+y^2+z^2}} \right) \hat{j} - \frac{2}{2z} \left( \frac{1}{\sqrt{x^2+y^2+z^2}} \right) \hat{k}$$

$$E = \frac{x}{(x^2+y^2+z^2)^{\frac{3}{2}}} \hat{i} + \frac{y}{(x^2+y^2+z^2)^{\frac{3}{2}}} \hat{j} + \frac{z}{(x^2+y^2+z^2)^{\frac{3}{2}}} \hat{k}$$

$$E = \frac{x \hat{i} + y \hat{j} + z \hat{k}}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$
2. Let $S$ be the portion of the paraboloid $x = y^2 + z^2$ for which $x \leq 9$ and $y \leq 0$, and assume that the density of $S$ is given by $\rho(x,y,z) = xz^2$. Set up, but do not evaluate, an integral to compute the mass of $S$. (Your answer should be a double integral of the kind you knew how to evaluate last term.)

\[
\mathbf{r}(\varphi, \theta) = u^2 \mathbf{i} + u \cos \varphi \mathbf{j} + u \sin \varphi \mathbf{k}
\]

\[
\left| \begin{array}{ccc} 1 & 0 & 0 \\ 0 & u \cos \varphi \sin \varphi & -u \sin \varphi \\ 0 & u \sin \varphi \cos \varphi & u \cos \varphi \end{array} \right| = u^4 - 2u^2 \cos \varphi \sin \varphi - 2u^2 \sin \varphi \cos \varphi
\]

\[
\left| \frac{\partial z}{\partial u} \times \frac{\partial z}{\partial \varphi} \right| = \sqrt{u^2 + 4u^4 \cos^2 \varphi + 4u^4 \sin^2 \varphi} = \sqrt{u^2 + 4u^4}
\]

\[
P = xz^2 = (u^2)(u^2 \sin^2 \varphi) = u^4 \sin^2 \varphi
\]

\[
M = \iiint_S \rho \, dS = \int_0^{\pi/2} \int_0^{\varepsilon} P \left| \frac{\partial z}{\partial u} \times \frac{\partial z}{\partial \varphi} \right| \, du \, dv
\]

\[
= \frac{v = \frac{3\pi}{2}}{v = \frac{\pi}{2}} \int_{u = 0}^{u = 3} u^4 \sin^2 \varphi \sqrt{u^2 + 4u^4} \, du \, dv
\]