Instructions:
1. Read all directions.
2. In keeping with the Union College policy on academic honesty, you should neither accept nor provide unauthorized assistance in the completion of this work.

Direction 1: Circle the correct answer

1. Two like charges
   a. attract due to coulomb's force.
   b. repel due to coulomb's force.
   c. neither attract nor repel due to coulomb's force.
   d. none of the above

2. Two electrons (e1 and e2) and a proton (p) lie on a straight line, as shown. The direction of the force of e2 on e1, the force of p on e1, and the total force on e1, respectively, are:
   a. $\rightarrow$, $\rightarrow$, $\rightarrow$
   b. $\leftarrow$, $\rightarrow$, $\rightarrow$
   c. $\rightarrow$, $\leftarrow$, $\leftarrow$
   d. $\leftarrow$, $\leftarrow$, $\leftarrow$
   e. $\leftarrow$, $\leftarrow$, $\leftarrow$

3. The units of electrical potential energy is/are
   a. J
   b. N/C
   c. m/s
   d. kg/m
   e. $\frac{J}{C}$

4. A free negative charge moves ___________ electric field due to the electrical force.
   a. in the same direction of the
   b. in the opposite direction of the
   c. perpendicular to the
   d. all of the above
   e. none of the above

5. A positive charge moves ___________ spontaneously (meaning on its own).
   a. from lower potential energy to higher potential energy.
   b. from higher potential energy to lower potential energy.
   c. all of the above.
   d. none of the above.
6. Match the vector field with its graph.

<table>
<thead>
<tr>
<th>vector field</th>
<th>graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-0.5x\hat{i} -0.5y\hat{j})</td>
<td>A B C D</td>
</tr>
<tr>
<td>(2\hat{i} - \hat{j})</td>
<td>A B C (D)</td>
</tr>
<tr>
<td>(0.3x\hat{j})</td>
<td>A B C D</td>
</tr>
<tr>
<td>(-y\hat{i} + x\hat{j})</td>
<td>A B C D</td>
</tr>
</tbody>
</table>

[Vector field graphs A, B, C, D shown]
Direction II: Solve the following Problems. In order to get full credit show all your work and justify your answer by reasoning.

7. Three point charges \((Q, q, 2Q)\) are placed on the x-axis as shown in the figure below. If \(Q = +10 \mu C, q = -5.0 \mu C,\) and \(d = 1.0m,\) what is the value of \(x_i\) (coordinate of \(q\)) so that the net electrical force experienced by \(q\) is zero?

\[
F = F_{Q} + F_{2Q} = 0 \quad \text{at} \quad x = x_i
\]

where \(x_i\) is the coordinate of \(q\).

\[
F_{Q} = \frac{Qq}{4\pi\varepsilon_0 x_i^2}
\]

\[
F_{2Q} = \frac{2Qq}{4\pi\varepsilon_0 (2-x_i)^2}
\]

\[
0 = \frac{Qq}{4\pi\varepsilon_0 x_i^2} + \frac{2Qq}{4\pi\varepsilon_0 (2-x_i)^2} \Rightarrow \frac{1}{x_i^2} - \frac{2}{(2-x_i)^2} = 0 \Rightarrow \frac{1}{x_i^2} = \frac{2}{(2-x_i)^2}
\]

\[
x_i = \frac{d}{\sqrt{2} + 1} \quad \text{or} \quad x_i = \frac{d}{\sqrt{2} - 1}
\]

So, \(x_i = \frac{1.0m}{\sqrt{2} + 1} = 0.414m\)

8. The curves below are level curves of the function \(f(x,y)\). At the point P, draw some vector that "could" be \(\nabla f\). (Hint: Make sure to get the direction correct. You are not given enough information to know the correct magnitude.)

9. Consider the half cylinder given by \(x^2 + z^2 = 25,\) for \(4 \leq y \leq 7\) and \(z \geq 0.\) Parameterize this surface. In other words, describe this surface by giving \(x, y,\) and \(z\) in terms of two other variables. Be sure to state where these two other variables go from and to.

\[
x = 5 \cos \theta
\]

\[
y = y
\]

\[
z = 5 \sin \theta
\]

\[
\theta \rightarrow \pi \rightarrow 2\pi
\]

\[
y \rightarrow y \rightarrow 7
\]
10. Suppose that I am standing on a mountain whose height, as a function of position in the xy plane, is given by \( h(x,y) = x^3y^2 + 5x - y \).

a. Determine the (xy) direction of the steepest uphill at the point where \( x=2 \) and \( y=3 \).

\[
\nabla h(2,3) = (2x^3y^2 + 5)\mathbf{i} + (2x^3y - 1)\mathbf{j}
\]

b. What is the slope (i.e., the rate of change of vertical with respect to horizontal) from the point (2,3) in the direction of the point (5,1)?

\[
D_a h(2,3) = \nabla h(2,3) \cdot \mathbf{u} = \frac{1}{\sqrt{13}} (3\mathbf{i} - 2\mathbf{j})
\]

\[
D_a h(2,3) = \frac{1}{\sqrt{13}} (3\mathbf{i} + 4\mathbf{j}) \cdot \frac{1}{\sqrt{13}} (3\mathbf{i} - 2\mathbf{j})
\]

\[
= \frac{1}{13} (33 - 94) = \frac{245}{\sqrt{13}}
\]

c. Notice that since \( h(2,3) = 79 \), the point (2,3,79) is on the surface. Find parametric equations for the line that is normal to the mountain at this point.

\[
\mathbf{z} = x^3y^2 + 5x - y
\]

Let \( f(x,y,z) = x^3y^2 + 5x - y \)

Surface is: \( f(x,y,z) = 0 \)

\[
\nabla f(x,y,z) = (3x^2y^2 + 5)\mathbf{i} + (2x^3y - 1)\mathbf{j} - \mathbf{k}
\]

\[
\nabla f(2,3,79) = 113\mathbf{i} + 47\mathbf{j} - \mathbf{k}
\]

Normal line: \( x = 2 + 113t \)

\[
y = 3 + 47t
\]

\[
z = 79 - t
\]

d. State a (xy) direction (by specifying a vector) so that if I move in that direction from the point (2,3,79), my instantaneous rate of change of vertical with respect to horizontal will be zero.

Vector normal to \( \nabla h(2,3) \):

\[
47\mathbf{i} - 113\mathbf{j}
\]

or

\[
-47\mathbf{i} + 113\mathbf{j}
\]
11. A dipole is placed in a region where there is a uniform external electric field, 
\[ \overrightarrow{E} = 3 \times 10^6 \frac{N}{C} \left( \hat{i} + \hat{j} \right) \]. The charges on the dipole are \( q \) and \(-q\), where \( q = 2 \mu C \). The positive charge of the dipole is located at \((5 \mu m, 0, 0)\) and the negative charge is \((-5 \mu m, 0, 0)\).

a. Calculate the dipole moment of the dipole. (remember this is a vector quantity).

\[
\overrightarrow{p} = \overrightarrow{qd}
\]
\[
\overrightarrow{d} = (5 + 5) \mu m \hat{i} = 10 \mu m \hat{i}
\]
\[
\overrightarrow{p} = (2 \times 10^{-6} C)(10^{-6} \mu m) \hat{i} = 2 \times 10^{-9} \text{cm} \hat{i}
\]

b. Calculate the torque experienced by the dipole.

\[
\tau = \overrightarrow{p} \times \overrightarrow{E} = \left| \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ 2 \times 10^{-6} \text{cm} & 0 & 0 \\ 3 \times 10^6 \frac{N}{C} & 0 & 0 \end{array} \right| = 3 \sqrt{2} \times 10^{-5} \text{Nm} \hat{k}
\]

b. Calculate the potential energy of the system.

\[
U = -\overrightarrow{p} \cdot \overrightarrow{E}
\]
\[
U = -\left( (2 \times 10^{-11} \text{C.m}) \cdot \left( \frac{3 \times 10^6 \text{N} \cdot \hat{i}}{\sqrt{2} \text{C}} \right) \right)
\]
\[
U = -3 \sqrt{2} \times 10^{-5} \text{J} \hat{k} = -3 \sqrt{2} \times 10^{-5} \text{J}
\]
12. Consider the vector field \( \mathbf{E}(x,y,z) = (3x^2y+3)i + (x^3+2yz^4)j + 4y^2z^3k \).

a. Show that \( \mathbf{E} \) is conservative by finding a scalar potential function for \( \mathbf{E} \).

Suppose \( \mathbf{E} = \nabla \varphi \). Then:

\[
\frac{\partial \varphi}{\partial x} = 3x^2y + 3 \quad \frac{\partial \varphi}{\partial y} = x^3 + 2yz^4 \quad \frac{\partial \varphi}{\partial z} = 4y^2z^3
\]

\[\varphi = x^3y + 3x + y^2z^4\]

b. Now suppose that \( \mathbf{E} \) gives the electric field vector at any point \((x,y,z)\) due to some distribution of electric charge. Some charge \( q \) is at the point \((2,3,-1)\). Compute the force on this charge due to the field \( \mathbf{E} \).

\[\text{Force} = q \cdot \mathbf{E} (2,3,-1) = q (39i + 14j - 36k)\]

c. Use your answer to part a to compute the work done by the electric field on a charge \( q \) as the charge moves from \((2,3,-1)\) to \((0,-2,1)\) along the path shown on the board.

\[
\text{Work} = \int_{(2,3,-1)}^{(0,-2,1)} \mathbf{F} \cdot d\mathbf{s} = q \int_{(2,3,-1)}^{(0,-2,1)} \mathbf{E} \cdot d\mathbf{s} = q \left[ \varphi (0,-2,1) - \varphi (2,3,-1) \right] = q [4 - (24 + 6 + 9)] = -35q\]
13. A rod is bent in the shape of a semi-circle as shown in the figure below. The center of the semi-circle coincides with the origin (i.e., Point O is the origin and is also the center of the semicircle). The linear charge density on the rod is \( \lambda = 5 \mu \text{C/m} \). The semi-circle has a radius of \( r = 5 \text{cm} \).

a. Find the total charge on the semi-circle.

\[ Q = \lambda \oint \, ds = \lambda \int_{0}^{\pi} r \, d\theta = 2 \pi r \lambda \]
\[ Q = (5 \times 10^{-8} \text{C/m}) \left( 5 \times 10^{-2} \text{m} \right) \left( \pi \right) = 25 \pi \times 10^{-8} \text{C} \approx 0.78 \mu \text{C} \]

b. Find the electric field at point O due to the charge on the semi-circle. (remember electric field is a vector)

The electric field due to a small charge \( dq \) of radius \( r \) is given by \( d\mathbf{E} \),

\[ d\mathbf{E} = \frac{1}{4 \pi \varepsilon_0} \frac{\lambda \, d\theta}{r^2} \mathbf{r} \]

where \( \mathbf{r} = \sin \theta \mathbf{i} - \cos \theta \mathbf{j} \). \( \mathbf{r} \) is the vector from the origin to a point on the semi-circle.

\[ d\mathbf{E} = \frac{1}{4 \pi \varepsilon_0} \frac{\lambda \, d\theta}{r^2} (\sin \theta \mathbf{i} - \cos \theta \mathbf{j}) \]

The net electric field is then

\[ \mathbf{E} = \int d\mathbf{E} = \frac{1}{4 \pi \varepsilon_0} \int_{0}^{\pi} \frac{\lambda \, d\theta}{r^2} \left[ (\sin \theta \mathbf{i}) - (\cos \theta \mathbf{j}) \right] \]

\[ \mathbf{E} = \frac{1}{4 \pi \varepsilon_0} \frac{\lambda}{r^2} \left[ (\sin 0 \mathbf{i}) - (\cos 0 \mathbf{j}) \right] = \frac{1}{4 \pi \varepsilon_0} \frac{\lambda}{r^2} \left[ 0 \mathbf{i} - 1 \mathbf{j} \right] \]

\[ \mathbf{E} = \frac{1}{4 \pi \varepsilon_0} \frac{2 \lambda}{r^2} \left[ -\mathbf{j} \right] = \frac{1}{4 \pi \varepsilon_0} \frac{2 \lambda}{r^2} \mathbf{j} \]

\[ \mathbf{E} = (9 \times 10^9 \text{N/C}^2) \frac{2 \left( 5 \times 10^{-8} \text{C/m} \right)}{r^2} \mathbf{j} = 1.8 \times 10^6 \mathbf{j} \text{N/C} \]