Homework: Solutions (Chapter 27)

1. (a) The energy transferred is

\[ U = Pt = \frac{\varepsilon^2 t}{r + R} = \frac{(2.0 \, \text{V})^2 (2.0 \, \text{min}) (60 \, \text{s/min})}{1.0 \, \Omega + 5.0 \, \Omega} = 80 \, \text{J}. \]

(b) The amount of thermal energy generated is

\[ U' = i^2 R t = \left( \frac{\varepsilon}{r + R} \right)^2 R t = \left( \frac{2.0 \, \text{V}}{1.0 \, \Omega + 5.0 \, \Omega} \right)^2 (5.0 \, \Omega) (2.0 \, \text{min}) (60 \, \text{s/min}) = 67 \, \text{J}. \]

(c) The difference between \( U \) and \( U' \), which is equal to 13 J, is the thermal energy that is generated in the battery due to its internal resistance.

7. (a) Let \( i \) be the current in the circuit and take it to be positive if it is to the left in \( R_1 \). We use Kirchhoff's loop rule: \( \varepsilon_1 - i R_2 - i R_1 - \varepsilon_2 = 0 \). We solve for \( i \):

\[ i = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{12 \, \text{V} - 6.0 \, \text{V}}{4.0 \, \Omega + 8.0 \, \Omega} = 0.50 \, \text{A}. \]

A positive value is obtained, so the current is counterclockwise around the circuit.

If \( i \) is the current in a resistor \( R \), then the power dissipated by that resistor is given by \( P = i^2 R \).

(b) For \( R_1 \), \( P_1 = i^2 R_1 = (0.50 \, \text{A})^2 (4.0 \, \Omega) = 1.0 \, \text{W} \),

(c) and for \( R_2 \), \( P_2 = i^2 R_2 = (0.50 \, \text{A})^2 (8.0 \, \Omega) = 2.0 \, \text{W} \).

If \( i \) is the current in a battery with emf \( \varepsilon \), then the battery supplies energy at the rate \( P = i \varepsilon \) provided the current and emf are in the same direction. The battery absorbs energy at the rate \( P = i \varepsilon \) if the current and emf are in opposite directions.

(d) For \( \varepsilon_1 \), \( P_1 = i \varepsilon_1 = (0.50 \, \text{A})(12 \, \text{V}) = 6.0 \, \text{W} \)

(e) and for \( \varepsilon_2 \), \( P_2 = i \varepsilon_2 = (0.50 \, \text{A})(6.0 \, \text{V}) = 3.0 \, \text{W} \).

(f) In battery 1 the current is in the same direction as the emf. Therefore, this battery supplies energy to the circuit; the battery is discharging.

(g) The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.
26. (a) \( R_{eq} (FH) = (10.0 \ \Omega)(10.0 \ \Omega)(5.00 \ \Omega)/[(10.0 \ \Omega)(10.0 \ \Omega) + 2(10.0 \ \Omega)(5.00 \ \Omega)] = 2.50 \ \Omega \).

(b) \( R_{eq} (FG) = (5.00 \ \Omega) R/(R + 5.00 \ \Omega) \), where

\[
R = 5.00 \ \Omega + (5.00 \ \Omega)(10.0 \ \Omega)/(5.00 \ \Omega + 10.0 \ \Omega) = 8.33 \ \Omega.
\]

So \( R_{eq} (FG) = (5.00 \ \Omega)(8.33 \ \Omega)/(5.00 \ \Omega + 8.33 \ \Omega) = 3.13 \ \Omega \).

31. First, we note \( V_4 \), that the voltage across \( R_4 \) is equal to the sum of the voltages across \( R_5 \) and \( R_6 \):

\[
V_4 = i_6 (R_5 + R_6) = (1.40 \ \text{A})(8.00 \ \Omega + 4.00 \ \Omega) = 16.8 \ \text{V}.
\]

The current through \( R_4 \) is then equal to \( i_4 = V_4/R_4 = 16.8 \ \text{V}/(16.0 \ \Omega) = 1.05 \ \text{A} \).

By the junction rule, the current in \( R_2 \) is

\[
i_2 = i_4 + i_6 = 1.05 \ \text{A} + 1.40 \ \text{A} = 2.45 \ \text{A}.
\]

so its voltage is \( V_2 = (2.00 \ \Omega)(2.45 \ \text{A}) = 4.90 \ \text{V} \).

The loop rule tells us the voltage across \( R_3 \) is \( V_3 = V_2 + V_4 = 21.7 \ \text{V} \) (implying that the current through it is \( i_3 = V_3/(2.00 \ \Omega) = 10.85 \ \text{A} \)).

The junction rule now gives the current in \( R_1 \) as \( i_1 = i_2 + i_3 = 2.45 \ \text{A} + 10.85 \ \text{A} = 13.3 \ \text{A} \), implying that the voltage across it is \( V_1 = (13.3 \ \text{A})(2.00 \ \Omega) = 26.6 \ \text{V} \). Therefore, by the loop rule,

\[
\varepsilon = V_1 + V_3 = 26.6 \ \text{V} + 21.7 \ \text{V} = 48.3 \ \text{V}.
\]

34. (a) The voltage across \( R_3 = 6.0 \ \Omega \) is \( V_3 = iR_3 = (6.0 \ \text{A})(6.0 \ \Omega) = 36 \ \text{V} \). Now, the voltage across \( R_1 = 2.0 \ \Omega \) is

\[
(V_4 - V_3) - V_3 = 78 - 36 = 42 \ \text{V},
\]

which implies the current is \( i_1 = (42 \ \text{V})/(2.0 \ \Omega) = 21 \ \text{A} \). By the junction rule, then, the current in \( R_2 = 4.0 \ \Omega \) is

\[
i_2 = i_1 - i = 21 \ \text{A} - 6.0 \ \text{A} = 15 \ \text{A}.
\]

The total power dissipated by the resistors is (using Eq. 26-27)

\[
i_1^2 (2.0 \ \Omega) + i_2^2 (4.0 \ \Omega) + i_3^2 (6.0 \ \Omega) = 1998 \ \text{W} \approx 2.0 \ \text{kW}.
\]

By contrast, the power supplied (externally) to this section is \( P_4 = i_4 (V_4 - V_3) \) where \( i_4 = i_1 = 21 \ \text{A} \). Thus, \( P_4 = 1638 \ \text{W} \). Therefore, the "Box" must be providing energy.

(b) The rate of supplying energy is \( (1998 - 1638) \ \text{W} = 3.6 \times 10^2 \ \text{W} \).
37. (a) We note that the \( R_1 \) resistors occur in series pairs, contributing net resistance \( 2R_1 \)
in each branch where they appear. Since \( \phi_2 = \phi_3 \) and \( R_2 = 2R_1 \), from symmetry we know
that the currents through \( \phi_2 \) and \( \phi_3 \) are the same: \( i_2 = i_3 = i \). Therefore, the current through
\( \phi_1 \) is \( i_1 = 2i \). Then from \( V_b - V_d = \phi_2 - iR_2 = \phi_1 + (2R_1)(2i) \) we get

\[
 i = \frac{\phi_1 - \phi_2}{4R_1 + R_2} = \frac{4.0 \text{ V} - 2.0 \text{ V}}{4(1.0 \text{ } \Omega) + 2.0 \text{ } \Omega} = 0.33 \text{ A.}
\]

Therefore, the current through \( \phi_1 \) is \( i_1 = 2i = 0.67 \text{ A.} \)

(b) The direction of \( i_1 \) is downward.

(c) The current through \( \phi_2 \) is \( i_2 = 0.33 \text{ A.} \)

(d) The direction of \( i_2 \) is upward.

(e) From part (a), we have \( i_3 = i_2 = 0.33 \text{ A.} \)

(f) The direction of \( i_3 \) is also upward.

(g) \( V_b - V_d = -iR_2 + \phi_2 = -(0.333 \text{ A})(2.0 \text{ } \Omega) + 4.0 \text{ V} = 3.3 \text{ V.} \)

61. Here we denote the battery emf as \( V \). Then the requirement stated in the problem that
the resistor voltage be equal to the capacitor voltage becomes \( iR = V_{\text{cap.}} \) or

\[
 V e^{-t/R \text{C}} = V (1 - e^{-t/R \text{C}})
\]

where Eqs. 27-34 and 27-35 have been used. This leads to \( t = R \text{C} \ln 2 \), or \( t = 0.208 \text{ ms.} \)

94. In the steady state situation, there is no current going to the capacitors, so the resistors
all have the same current. By the loop rule,

\[
 20.0 \text{ V} = (5.00 \text{ } \Omega)i + (10.0 \text{ } \Omega)i + (15.0 \text{ } \Omega)i
\]

which yields \( i = \frac{2}{3} \text{ A.} \). Consequently, the voltage across the \( R_1 = 5.00 \text{ } \Omega \) resistor is \( (5.00 \text{ } \Omega)(2/3 \text{ A}) = 10/3 \text{ V,} \) and is equal to the voltage \( V_1 \) across the \( C_1 = 5.00 \text{ } \mu \text{F} \) capacitor. Using Eq. 26-22, we find the stored energy on that capacitor:

\[
 U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} (5.00 \times 10^{-6} \text{ F})(\frac{10}{3} \text{ V})^2 = 2.78 \times 10^{-5} \text{ J.}
\]

Similarly, the voltage across the \( R_2 = 10.0 \text{ } \Omega \) resistor is \( (10.0 \text{ } \Omega)(2/3 \text{ A}) = 20/3 \text{ V} \) and is
equal to the voltage \( V_2 \) across the \( C_2 = 10.0 \text{ } \mu \text{F} \) capacitor. Hence,

\[
 U_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2} (10.0 \times 10^{-6} \text{ F})(\frac{20}{3} \text{ V})^2 = 2.22 \times 10^{-5} \text{ J}
\]

Therefore, the total capacitor energy is \( U_1 + U_2 = 2.50 \times 10^{-4} \text{ J.} \)