2. We use \( \Phi = \int \vec{E} \cdot d\vec{A} \) and note that the side length of the cube is \((3.0 \text{ m} - 1.0 \text{ m}) = 2.0 \text{ m} \).

(a) On the top face of the cube \( y = 2.0 \text{ m} \) and \( d\vec{A} = (dA) \hat{j} \). Therefore, we have
\[
\vec{E} = 4\hat{i} - 3\left( \left(2.0\right)^2 + 2 \right)\hat{j} = 4\hat{i} - 18\hat{j}
\]
Thus the flux is
\[
\Phi = \int_{\text{top}} \vec{E} \cdot d\vec{A} = \int_{\text{top}} (4\hat{i} - 18\hat{j}) \cdot (dA) \hat{j} = -18 \int_{\text{top}} dA = (-18)(2.0)^2 \text{ N} \cdot \text{m}^2 / \text{C} = -72 \text{ N} \cdot \text{m}^2 / \text{C}.
\]

(b) On the bottom face of the cube \( y = 0 \) and \( d\vec{A} = (dA)(-\hat{j}) \). Therefore, we have
\[
\vec{E} = 4\hat{i} - 3(0^2 + 2)\hat{j} = 4\hat{i} - 6\hat{j}
\]
Thus, the flux is
\[
\Phi = \int_{\text{bottom}} \vec{E} \cdot d\vec{A} = \int_{\text{bottom}} (4\hat{i} - 6\hat{j}) \cdot (dA)(-\hat{j}) = 6 \int_{\text{bottom}} dA = 6(2.0)^2 \text{ N} \cdot \text{m}^2 / \text{C} = +24 \text{ N} \cdot \text{m}^2 / \text{C}.
\]

(c) On the left face of the cube \( d\vec{A} = (dA)(-\hat{i}) \). So
\[
\Phi = \int_{\text{left}} \vec{E} \cdot d\vec{A} = \int_{\text{left}} (4\hat{i} + E_y\hat{j}) \cdot (dA)(-\hat{i}) = -4 \int_{\text{bottom}} dA = -4(2.0)^2 \text{ N} \cdot \text{m}^2 / \text{C} = -16 \text{ N} \cdot \text{m}^2 / \text{C}.
\]

(d) On the back face of the cube \( d\vec{A} = (dA)(-\hat{k}) \). But since \( \vec{E} \) has no \( z \) component
\[
\vec{E} \cdot d\vec{A} = 0.
\]
Thus, \( \Phi = 0 \).

(e) We now have to add the flux through all six faces. One can easily verify that the flux through the front face is zero, while that through the right face is the opposite of that through the left one, or \(+16 \text{ N} \cdot \text{m}^2 / \text{C} \). Thus the net flux through the cube is
\[
\Phi = (-72 + 24 - 16 + 0 + 0 + 16) \text{ N} \cdot \text{m}^2 / \text{C} = -48 \text{ N} \cdot \text{m}^2 / \text{C}.
\]

4. There is no flux through the sides, so we have two “inward” contributions to the flux, one from the top (of magnitude \((34)(3.0)^2\)) and one from the bottom (of magnitude \((20)(3.0)^2\)). With “inward” flux being negative, the result is \( \Phi = -486 \text{ N} \cdot \text{m}^2 / \text{C} \). Gauss’ law then leads to
\[
q_{\text{enc}} = \varepsilon_0 \Phi = (8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2)(-486 \text{ N} \cdot \text{m}^2 / \text{C}) = -4.3 \times 10^{-9} \text{ C}.
\]

7. To exploit the symmetry of the situation, we imagine a closed Gaussian surface in the shape of a cube, of edge length \( d \), with a proton of charge \( q = +1.6 \times 10^{-19} \text{ C} \) situated at the inside center of the cube. The cube has six faces, and we expect an equal amount of flux through each face. The total amount of flux is \( \Phi_{\text{net}} = q / \varepsilon_0 \), and we conclude that the flux through the square is one-sixth of that. Thus,
\[
\Phi = \frac{q}{6 \varepsilon_0} = \frac{1.6 \times 10^{-19} \text{ C}}{6(8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2)} = 3.01 \times 10^{-9} \text{ N} \cdot \text{m}^2 / \text{C}.
\]
36. The charge distribution in this problem is equivalent to that of an infinite sheet of charge with surface charge density \( \sigma = 4.50 \times 10^{-12} \text{ C/m}^2 \) plus a small circular pad of radius \( R = 1.80 \text{ cm} \) located at the middle of the sheet with charge density \(-\sigma\). We denote the electric fields produced by the sheet and the pad with subscripts 1 and 2, respectively. Using Eq. 22-26 for \( \vec{E}_2 \), the net electric field \( \vec{E} \) at a distance \( z = 2.56 \text{ cm} \) along the central axis is then

\[
\vec{E} = \vec{E}_1 + \vec{E}_2 = \left( \frac{\sigma}{2\varepsilon_0} \right) \hat{k} + \left( -\frac{\sigma}{2\varepsilon_0} \right) \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right) \hat{k} = \frac{\sigma z}{2\varepsilon_0 \sqrt{z^2 + R^2}} \hat{k}
\]

\[
= \frac{(4.50 \times 10^{-12} \text{ C/m}^2)(2.56 \times 10^{-2} \text{ m})}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)\sqrt{(2.56 \times 10^{-2} \text{ m})^2 + (1.80 \times 10^{-2} \text{ m})^2}} \hat{k} = (0.208 \text{ N/C}) \hat{k}
\]

41. The forces acting on the ball are shown in the diagram on the right. The gravitational force has magnitude \( mg \), where \( m \) is the mass of the ball; the electrical force has magnitude \( qE \), where \( q \) is the charge on the ball and \( E \) is the magnitude of the electric field at the position of the ball; and, the tension in the thread is denoted by \( T \). The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle \( \theta \) (= 30°) with the vertical.

Since the ball is in equilibrium the net force on it vanishes. The sum of the horizontal components yields

\[
qE - T \sin \theta = 0
\]

and the sum of the vertical components yields

\[
T \cos \theta - mg = 0.
\]

The expression \( T = qE/\sin \theta \), from the first equation, is substituted into the second to obtain \( qE = mg \tan \theta \). The electric field produced by a large uniform plane of charge is given by \( E = \sigma/2\varepsilon_0 \), where \( \sigma \) is the surface charge density. Thus,

\[
\frac{q\sigma}{2\varepsilon_0} = mg \tan \theta
\]

and

\[
\sigma = \frac{2\varepsilon_0 mg \tan \theta}{q} = \frac{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.0 \times 10^{-6} \text{ kg})(9.8 \text{ m/s}^2)}{2.0 \times 10^{-8} \text{ C}} \tan 30^\circ
\]

\[
= 5.0 \times 10^{-9} \text{ C/m}^2.
\]
50. The field is zero for \(0 \leq r \leq a\) as a result of Eq. 23-16. Thus,

(a) \(E = 0\) at \(r = 0\),

(b) \(E = 0\) at \(r = a/2.00\), and

(c) \(E = 0\) at \(r = a\).

For \(a \leq r \leq b\) the enclosed charge \(q_{enc}\) (for \(a \leq r \leq b\)) is related to the volume by

\[
q_{enc} = \rho \left( \frac{4\pi r^3}{3} - \frac{4\pi a^3}{3} \right).
\]

Therefore, the electric field is

\[
E = \frac{1}{4\pi \varepsilon_0} \frac{q_{enc}}{r^2} = \frac{\rho}{4\pi \varepsilon_0 r^2} \left( \frac{4\pi r^3}{3} - \frac{4\pi a^3}{3} \right) = \frac{\rho}{3\varepsilon_0} \frac{r^3 - a^3}{r^2}
\]

for \(a \leq r \leq b\).

(d) For \(r = 1.50a\), we have

\[
E = \frac{\rho}{3\varepsilon_0} \frac{(1.50a)^3 - a^3}{(1.50a)^2} = \frac{\rho a}{3\varepsilon_0} \left( \frac{2.375}{2.25} \right) = \frac{(1.84 \times 10^{-9} \text{ C/m}^3)(0.100 \text{ m})}{3(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} \left( \frac{2.375}{2.25} \right) = 7.32 \text{ N/C}.
\]

(e) For \(r = b = 2.00a\), the electric field is

\[
E = \frac{\rho}{3\varepsilon_0} \frac{(2.00a)^3 - a^3}{(2.00a)^2} = \frac{\rho a}{3\varepsilon_0} \left( \frac{7}{4} \right) = \frac{(1.84 \times 10^{-9} \text{ C/m}^3)(0.100 \text{ m})}{3(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} \left( \frac{7}{4} \right) = 12.1 \text{ N/C}.
\]

(f) For \(r \geq b\) we have \(E = q_{total} / 4\pi \varepsilon_0 r^2\) or

\[
E = \frac{\rho}{3\varepsilon_0} \frac{b^3 - a^3}{r^2}.
\]

Thus, for \(r = 3.00b = 6.00a\), the electric field is

\[
E = \frac{\rho}{3\varepsilon_0} \frac{(2.00a)^3 - a^3}{(6.00a)^2} = \frac{\rho a}{3\varepsilon_0} \left( \frac{7}{36} \right) = \frac{(1.84 \times 10^{-9} \text{ C/m}^3)(0.100 \text{ m})}{3(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} \left( \frac{7}{36} \right) = 1.35 \text{ N/C}.
\]
51. At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so \( \oint \vec{E} \cdot d\vec{A} = 4\pi r^2 E \), where \( r \) is the radius of the Gaussian surface.

For \( r < a \), the charge enclosed by the Gaussian surface is \( q_1(r/a)^3 \). Gauss’ law yields

\[
4\pi r^2 E = \left( \frac{q_1}{\varepsilon_0} \right) \left( \frac{r}{a} \right)^3 \Rightarrow E = \frac{q_1 r}{4\pi\varepsilon_0 a^3}.
\]

(a) For \( r = 0 \), the above equation implies \( E = 0 \).

(b) For \( r = a/2 \), we have

\[
E = \frac{q_1 (a/2)}{4\pi\varepsilon_0 a^3} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-15} \text{ C})}{2(2.00 \times 10^{-2} \text{ m})^2} = 5.62 \times 10^{-2} \text{ N/C}.
\]

(c) For \( r = a \), we have

\[
E = \frac{q_1}{4\pi\varepsilon_0 a^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-15} \text{ C})}{(2.00 \times 10^{-2} \text{ m})^2} = 0.112 \text{ N/C}.
\]

In the case where \( a < r < b \), the charge enclosed by the Gaussian surface is \( q_1 \), so Gauss’ law leads to

\[
4\pi r^2 E = \frac{q_1}{\varepsilon_0} \Rightarrow E = \frac{q_1}{4\pi\varepsilon_0 r^2}.
\]

(d) For \( r = 1.50a \), we have

\[
E = \frac{q_1}{4\pi\varepsilon_0 (1.50a)^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-15} \text{ C})}{(1.50 \times 2.00 \times 10^{-2} \text{ m})^2} = 0.0499 \text{ N/C}.
\]

(e) In the region \( b < r < c \), since the shell is conducting, the electric field is zero. Thus, for \( r = 2.30a \), we have \( E = 0 \).

(f) For \( r > c \), the charge enclosed by the Gaussian surface is zero. Gauss’ law yields

\[
4\pi r^2 E = 0 \Rightarrow E = 0.
\]

Thus, \( E = 0 \) at \( r = 3.50a \).

(g) Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface, \( \oint \vec{E} \cdot d\vec{A} = 0 \) and, according to Gauss’ law, the net charge enclosed by the surface is zero. If \( Q_i \) is the charge on the inner surface of the shell, then \( q_1 + Q_i = 0 \) and \( Q_i = -q_1 = -5.00 \text{ fC} \).

(h) Let \( Q_o \) be the charge on the outer surface of the shell. Since the net charge on the shell is \( -q \), \( Q_i + Q_o = -q_1 \). This means

\[
Q_o = -q_1 - Q_i = -q_1 - (-q_1) = 0.
\]