Homework #6 Solutions  Chapter 24

12. (a) The potential difference is

\[ V_A - V_B = \frac{q}{4\pi \varepsilon_0 r_A} - \frac{q}{4\pi \varepsilon_0 r_B} = (1.0 \times 10^{-5} \text{ C})(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(\frac{1}{2.0 \text{ m}} - \frac{1}{1.0 \text{ m}}) \]

\[ = -4.5 \times 10^3 \text{ V.} \]

(b) Since \( V(r) \) depends only on the magnitude of \( \vec{r} \), the result is unchanged.

13. (a) The charge on the sphere is

\[ q = 4\pi \varepsilon_0 VR = \frac{(200 \text{ V})(0.15 \text{ m})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 3.3 \times 10^{-9} \text{ C.} \]

(b) The (uniform) surface charge density (charge divided by the area of the sphere) is

\[ \sigma = \frac{q}{4\pi R^2} = \frac{3.3 \times 10^{-9} \text{ C}}{4\pi (0.15 \text{ m})^2} = 1.2 \times 10^{-8} \text{ C} / \text{m}^2. \]

23. (a) All the charge is the same distance \( R \) from \( C \), so the electric potential at \( C \) is

\[ V = \frac{1}{4\pi \varepsilon_0} \left( \frac{Q_1}{R} - \frac{6Q_1}{R} \right) = -\frac{5Q_1}{4\pi \varepsilon_0 R} = \frac{5(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(4.20 \times 10^{-12} \text{ C})}{8.20 \times 10^{-2} \text{ m}} = -2.30 \text{ V,} \]

where the zero was taken to be at infinity.

(b) All the charge is the same distance from \( P \). That distance is \( \sqrt{R^2 + D^2} \), so the electric potential at \( P \) is

\[ V = \frac{1}{4\pi \varepsilon_0} \left[ \frac{Q_1}{\sqrt{R^2 + D^2}} - \frac{6Q_1}{\sqrt{R^2 + D^2}} \right] = -\frac{5Q_1}{4\pi \varepsilon_0 \sqrt{R^2 + D^2}} \]

\[ = \frac{5(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(4.20 \times 10^{-12} \text{ C})}{\sqrt{(8.20 \times 10^{-2} \text{ m})^2 + (6.71 \times 10^{-2} \text{ m})^2}} = -1.78 \text{ V.} \]

25. (a) From Eq. 24-35, we find the potential to be

\[ V = 2 \cdot \frac{\lambda}{4\pi \varepsilon_0} \ln \left[ \frac{L/2 + \sqrt{(L^2/4) + d^2}}{d} \right] \]

\[ = 2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.68 \times 10^{-12} \text{ C/m}) \ln \left[ \frac{(0.06 \text{ m} / 2) + \sqrt{(0.06 \text{ m})^2/4 + (0.08 \text{ m})^2}}{0.08 \text{ m}} \right] \]

\[ = 2.43 \times 10^{-2} \text{ V.} \]

(b) The potential at \( P \) is \( V = 0 \) due to superposition.
28. The dipole potential is given by Eq. 24-30 (with $\theta = 90^\circ$ in this case)

$$V = \frac{p \cos \theta}{4\pi \varepsilon_0 r^2} = \frac{p \cos 90^\circ}{4\pi \varepsilon_0 r^2} = 0$$

since $\cos(90^\circ) = 0$. The potential due to the short arc is $q_1 / 4\pi \varepsilon_0 r_1$ and that caused by the long arc is $q_2 / 4\pi \varepsilon_0 r_2$. Since $q_1 = +2 \mu C$, $r_1 = 4.0 \text{ cm}$, $q_2 = -3 \mu C$, and $r_2 = 6.0 \text{ cm}$, the potentials of the arcs cancel. The result is zero.

35. We use Eq. 24-41:

$$E_x(x, y) = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x} \left((2.0 \text{ V/m}^2) x^2 - 3.0 \text{ V/m}^2) y^2\right) = -2(2.0 \text{ V/m}^2)x;$$

$$E_y(x, y) = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y} \left((2.0 \text{ V/m}^2) x^2 - 3.0 \text{ V/m}^2) y^2\right) = 2(3.0 \text{ V/m}^2)y.$$

We evaluate at $x = 3.0 \text{ m}$ and $y = 2.0 \text{ m}$ to obtain

$$\vec{E} = (-12 \text{ V/m})\hat{i} + (12 \text{ V/m})\hat{j}.$$

37. The electric field (along some axis) is the (negative of the) derivative of the potential $V$ with respect to the corresponding coordinate. In this case, the derivatives can be read off of the graphs as slopes (since the graphs are of straight lines). Thus,

$$E_x = -\frac{\partial V}{\partial x} = \left(\frac{-500 \text{ V}}{0.20 \text{ m}}\right) = -2500 \text{ V/m} = -2500 \text{ N/C}$$

$$E_y = -\frac{\partial V}{\partial y} = \left(\frac{300 \text{ V}}{0.30 \text{ m}}\right) = -1000 \text{ V/m} = -1000 \text{ N/C}.$$

These components imply the electric field has a magnitude of 2693 N/C and a direction of $-21.8^\circ$ (with respect to the positive $x$ axis). The force on the electron is given by $\vec{F} = q\vec{E}$ where $q = -e$. The minus sign associated with the value of $q$ has the implication that $\vec{F}$ points in the opposite direction from $\vec{E}$ (which is to say that its angle is found by adding 180° to that of $\vec{E}$). With $e = 1.60 \times 10^{-19} \text{ C}$, we obtain

$$\vec{F} = (-1.60 \times 10^{-19} \text{ C})[(2500 \text{ N/C})\hat{i} - (1000 \text{ N/C})\hat{j}] = (-4.0 \times 10^{-16} \text{ N})\hat{i} + (1.60 \times 10^{-16} \text{ N})\hat{j}.$$