Homework: Solutions (Chapter 29)

1. (a) The field due to the wire, at a point 8.0 cm from the wire, must be 39 μT and must be directed due south. Since \( B = \frac{\mu_0 i}{2\pi r} \),

\[
i = \frac{2\pi r B}{\mu_0} = \frac{2\pi (0.080 \text{ m})(39 \times 10^{-6} \text{ T})}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} = 16 \text{ A.}
\]

(b) The current must be from west to east to produce a field which is directed southward at points below it.

5. (a) Recalling the straight sections discussion in Sample Problem 29-1, we see that the current in the straight segments collinear with \( P \) do not contribute to the field at that point. Using Eq. 29-9 (with \( \phi = \theta \)) and the right-hand rule, we find that the current in the semicircular arc of radius \( b \) contributes \( \mu_0 i \phi / 4\pi b \) (out of the page) to the field at \( P \). Also, the current in the large radius arc contributes \( \mu_0 i \phi / 4\pi a \) (into the page) to the field there. Thus, the net field at \( P \) is

\[
B = \frac{\mu_0 i \phi}{4} \left( \frac{1}{b} - \frac{1}{a} \right) = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(0.411 \text{ A})(74^\circ \cdot \pi/180^\circ)}{4\pi} \left( \frac{1}{0.107 \text{ m}} - \frac{1}{0.135 \text{ m}} \right)
\]

\[
= 1.02 \times 10^{-7} \text{ T.}
\]

(b) The direction is out of the page.

8. (a) Recalling the straight sections discussion in Sample Problem 29-1, we see that the current in the straight segments collinear with \( C \) do not contribute to the field at that point.

Eq. 29-9 (with \( \phi = \pi \)) indicates that the current in the semicircular arc contributes \( \mu_0 i / 4R \) to the field at \( C \). Thus, the magnitude of the magnetic field is

\[
B = \frac{\mu_0 i}{4R} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(0.0348 \text{ A})}{4(0.0926 \text{ m})} = 1.18 \times 10^{-7} \text{ T.}
\]

(b) The right-hand rule shows that this field is into the page.

9. (a) \( B_{R_1} = \mu_0 i_1 / 2\pi r_1 \) where \( i_1 = 6.5 \text{ A} \) and \( r_1 = d_1 + d_2 = 0.75 \text{ cm} + 1.5 \text{ cm} = 2.25 \text{ cm} \), and \( B_{R_2} = \mu_0 i_2 / 2\pi r_2 \) where \( r_2 = d_2 = 1.5 \text{ cm} \). From \( B_{P1} = B_{P2} \) we get

\[
i_2 = i_1 \left( \frac{r_2}{r_1} \right) = (6.5 \text{ A}) \left( \frac{1.5 \text{ cm}}{2.25 \text{ cm}} \right) = 4.3 \text{ A.}
\]

(b) Using the right-hand rule, we see that the current \( i_2 \) carried by wire 2 must be out of the page.
28. (a) The contribution to \( B_C \) from the (infinite) straight segment of the wire is

\[
B_{C1} = \frac{\mu_0 j}{2\pi R}.
\]

The contribution from the circular loop is \( B_{C2} = \frac{\mu_0 j}{2R} \). Thus,

\[
B_C = B_{C1} + B_{C2} = \frac{\mu_0 j}{2R} \left(1 + \frac{1}{\pi}\right) = \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right) \left(5.78 \times 10^{-3} \text{ A}\right)}{2(0.0189 \text{ m})} \left(1 + \frac{1}{\pi}\right) = 2.53 \times 10^{-7} \text{ T}.
\]

\( B_C \) points out of the page, or in the +z direction. In unit-vector notation, \( \vec{B}_C = (2.53 \times 10^{-7} \text{ T}) \hat{k} \).

(b) Now \( \vec{B}_{C1} \perp \vec{B}_{C2} \) so

\[
B_C = \sqrt{B_{C1}^2 + B_{C2}^2} = \frac{\mu_0 j}{2R} \sqrt{1 + \frac{1}{\pi^2}} = \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right) \left(5.78 \times 10^{-3} \text{ A}\right)}{2(0.0189 \text{ m})} \sqrt{1 + \frac{1}{\pi^2}} = 2.02 \times 10^{-7} \text{ T}.
\]

and \( \vec{B}_C \) points at an angle (relative to the plane of the paper) equal to

\[
\tan^{-1} \left(\frac{B_{C1}}{B_{C2}}\right) = \tan^{-1} \left(\frac{1}{\pi}\right) = 17.66^\circ.
\]

In unit-vector notation,

\[
\vec{B}_C = 2.02 \times 10^{-7} \text{ T}(\cos 17.66^\circ \hat{i} + \sin 17.66^\circ \hat{k}) = (1.92 \times 10^{-7} \text{ T}) \hat{i} + (6.12 \times 10^{-8} \text{ T}) \hat{k}
\]

49. It is possible (though tedious) to use Eq. 29-26 and evaluate the contributions (with the intent to sum them) of all 200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 29-23 for the ideal solenoid (which does not make use of the coil diameter) is the preferred method:

\[
B = \mu_0 j m = \mu_0 j \left(\frac{N}{\ell}\right)
\]

where \( i = 0.30 \text{ A}, \ \ell = 0.25 \text{ m} \) and \( N = 200 \). This yields \( B = 3.0 \times 10^{-4} \text{ T} \).
81. (a) For the circular path \( L \) of radius \( r \) concentric with the conductor

\[
\oint_{L} \mathbf{B} \cdot d\mathbf{s} = 2\pi r B = \mu_{0} \phi_{\text{ext}} = \mu_{0} \frac{\pi (r^2 - b^2)}{\pi (a^2 - b^2)}.
\]

Thus, \( B = \frac{\mu_{0} j}{2\pi(a^2 - b^2)} \left( \frac{r^2 - b^2}{r} \right) \).

(b) At \( r = a \), the magnetic field strength is

\[
\frac{\mu_{0} j}{2\pi(a^2 - b^2)} \left( \frac{a^2 - b^2}{a} \right) = \frac{\mu_{0} j}{2\pi a}.
\]

At \( r = b \), \( B \propto r^2 - b^2 = 0 \). Finally, for \( b = 0 \)

\[
B = \frac{\mu_{0} j}{2\pi a^2} r = \frac{\mu_{0} j r}{2\pi a^2}
\]

which agrees with Eq. 29-20.

(c) The field is zero for \( r < b \) and is equal to Eq. 29-17 for \( r > a \), so this along with the result of part (a) provides a determination of \( B \) over the full range of values. The graph (with SI units understood) is shown below.