Homework: Solutions (Chapter 26)

1. (a) The charge that passes through any cross section is the product of the current and time. Since \( t = 4.0 \text{ min} = (4.0 \text{ min})(60 \text{ s/min}) = 240 \text{ s} \),

\[ q = it = (5.0 \text{ A})(240 \text{ s}) = 1.2 \times 10^3 \text{ C}. \]

(b) The number of electrons \( N \) is given by \( q = Ne \), where \( e \) is the magnitude of the charge on an electron. Thus,

\[ N = \frac{q}{e} = \frac{(1200 \text{ C})}{(1.60 \times 10^{-19} \text{ C})} = 7.5 \times 10^{21}. \]

8. (a) Circular area depends, of course, on \( r^2 \), so the horizontal axis of the graph in Fig. 26-24(b) is effectively the same as the area (enclosed at variable radius values), except for a factor of \( \pi \). The fact that the current increases linearly in the graph means that \( i/A = J = \text{constant} \). Thus, the answer is “yes, the current density is uniform.”

(b) We find \( i/(\pi r^2) = (0.005 \text{ A})/(\pi \times 4 \times 10^{-6} \text{ m}^2) = 398 \approx 4.0 \times 10^2 \text{ A/m}^2 \).

17. The resistance of the wire is given by \( R = \rho L / A \), where \( \rho \) is the resistivity of the material, \( L \) is the length of the wire, and \( A \) is its cross-sectional area. In this case,

\[ A = \pi r^2 = \pi (0.50 \times 10^{-3} \text{ m})^2 = 7.85 \times 10^{-7} \text{ m}^2. \]

Thus,

\[ \rho = \frac{RA}{L} = \frac{(50 \times 10^{-3} \Omega)(7.85 \times 10^{-7} \text{ m}^2)}{2.0 \text{ m}} = 2.0 \times 10^{-8} \Omega \cdot \text{m}. \]

22. (a) Since the material is the same, the resistivity \( \rho \) is the same, which implies (by Eq. 26-11) that the electric fields (in the various rods) are directly proportional to their current-densities. Thus, \( J_1 : J_2 : J_3 \) are in the ratio \( 2.5/4/1.5 \) (see Fig. 26-25). Now the currents in the rods must be the same (they are “in series”) so

\[ J_1 A_1 = J_3 A_3, \quad J_2 A_2 = J_3 A_3. \]

Since \( A = \pi r^2 \) this leads (in view of the aforementioned ratios) to

\[ 4r_2^2 = 1.5r_3^2, \quad 2.5r_1^2 = 1.5r_3^2. \]

Thus, with \( r_3 = 2 \text{ mm} \), the latter relation leads to \( r_1 = 1.55 \text{ mm} \).

(b) The \( 4r_2^2 = 1.5r_3^2 \) relation leads to \( r_2 = 1.22 \text{ mm} \).
26. Let \( r = 2.00 \text{ mm} \) be the radius of the kite string and \( t = 0.50 \text{ mm} \) be the thickness of the water layer. The cross-sectional area of the layer of water is

\[
A = \pi [(r + t)^2 - r^2] = \pi [(2.50 \times 10^{-3} \text{ m})^2 - (2.00 \times 10^{-3} \text{ m})^2] = 7.07 \times 10^{-6} \text{ m}^2.
\]

Using Eq. 26-16, the resistance of the wet string is

\[
R = \frac{\rho L}{A} = \frac{(150 \Omega \cdot \text{m})(800 \text{ m})}{7.07 \times 10^{-6} \text{ m}^2} = 1.698 \times 10^{10} \Omega.
\]

The current through the water layer is

\[
i = \frac{V}{R} = \frac{1.60 \times 10^8 \text{V}}{1.698 \times 10^{10} \Omega} = 9.42 \times 10^{-3} \text{A}.
\]

34. We follow the procedure used in Sample Problem 26-5.

Since the current spreads uniformly over the hemisphere, the current density at any given radius \( r \) from the striking point is \( J = I / 2\pi r^2 \). From Eq. 26-10, the magnitude of the electric field at a radial distance \( r \) is

\[
E = \rho_e J = \frac{\rho_e I}{2\pi r^2},
\]

where \( \rho_e = 30 \Omega \cdot \text{m} \) is the resistivity of water. The potential difference between a point at radial distance \( D \) and a point at \( D + \Delta r \) is

\[
\Delta V = -\int_D^{D+\Delta r} E \, dr = -\int_D^{D+\Delta r} \frac{\rho_e I}{2\pi r^2} \, dr = \frac{\rho_e I}{2\pi} \left( \frac{1}{D+\Delta r} - \frac{1}{D} \right) = -\frac{\rho_e I}{2\pi} \frac{\Delta r}{D(D+\Delta r)},
\]

which implies that the current across the swimmer is

\[
i = \frac{\Delta V}{R} = \frac{\rho_e I}{2\pi R D(D+\Delta r)}.
\]

Substituting the values given, we obtain

\[
i = \frac{(30.0 \Omega \cdot \text{m})(7.80 \times 10^4 \text{A})}{2\pi(4.00 \times 10^5 \Omega)}\left( \frac{0.70 \text{ m}}{(35.0 \text{ m})(35.0 \text{ m} + 0.70 \text{ m})} \right) = 5.22 \times 10^{-2} \text{A}.
\]

42. The resistance is \( R = P/i^2 = (100 \text{ W})/(3.00 \text{ A})^2 = 11.1 \Omega \).

54. From \( P = V^2 / R \), we have \( R = (5.0 \text{ V})^2/(200 \text{ W}) = 0.125 \Omega \). To meet the conditions of the problem statement, we must therefore set

\[
\int_0^L 5.00x \, dx = 0.125 \Omega
\]

Thus,

\[
\frac{5}{2} L^2 = 0.125 \quad \Rightarrow \quad L = 0.224 \text{ m}.
\]